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DRSAR/SA/N-70

SYSTEMS ANALYSIS DIRECTORATE

ACTIVITIES SUMMARY

JULY 1977

AUGUST 1977

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US ARMY ARMAMENT MATERIEL READINESS COMMAND

SYSTEMS ANALYSIS DIRECTORATE

ROCK ISLAND, ILLINOIS 61201

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

M44 Periscope PIP	Night Vision Devices	M110A1 SP Howitzer
M32E1 Periscope	Sheridan M551	M188E1 Propelling Charge
	XM204 Howitzer	M101A1 and M102 Howitzers

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This monthly publication contains Memoranda for Record (MFR's) and other technical information that summarize the activities of the Systems Analysis Directorate, US Army Materiel Readiness Command, Rock Island, IL (The most significant MFR's and other data will be published as notes or reports at a later date.)
The subjects dealt with are M44 Periscope, XM204 Howitzer, and M110A1 SP Howitzer.

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*Memorandum for Record and other technical information are grouped according to subject, when applicable, and in chronological order.

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DRSAR-SAA

MEMORANDUM FOR RECORD: 21 JUL 1977

SUBJECT: Cost Analysis of M44 Periscope Product Improvement Proposals

1. The purpose of this study is to determine the most cost effective product improvement proposal (PIP) for the M44 periscope. The alternatives considered were:

alternative 1. Continue with the existing periscope design.

alternative 2. Improve the prism bonding area, convert power supply, install a reset relay to prevent image intensifier tube damage and improve the sealing and purging system.

alternative 3. Modify the M44 periscope body and use the M32E1 night vision elbow.

alternative 4. Replace the M44 periscope with the M32E1 periscope.

2. The M44 Periscope was the first periscope to use the passive night vision image tube. Because of the length of this first generation tube, periscope designers were forced to utilize a complex optical design. Poor maintainability because of the complexity has resulted in high failure rates and high logistic support costs. Alternative 2 proposes to correct these problems by improving the existing M44 design. Advancements in the design of night vision devices have occurred since the M44 periscope was fielded. Second generation image intensifier tubes are more compact and have self-contained optical alignment and electrical controls. These advancements have enabled the night vision portions of the newer passive periscopes to be easily replaceable modules. One of these designs is the M32E1 periscope. The third alternative proposes to modify the M44 body to accept the M32E1 night vision elbow. The fourth alternative proposes to replace the entire M44 periscope with the M32E1 periscope. This alternative requires modification of the hull as the openings for the M44 and the M32E1 are not the same size.

3. The cost of ownership was determined for each of the alternatives. It was assumed that each alternative had been implemented at the beginning of the time period. It was further assumed that 1575 Sheridans would be in use. The one time PIP cost and the yearly maintenance cost was determined using the following equation.

$$Y = PT(B\ (B\$) + R\ (R\$))$$

where:

- P = fraction of periscopes replaced due to usage
- B = fraction of replaced items purchased
- B$ = purchase cost for one periscope
- R = fraction of units rebuilt
- R$ = cost of rebuilding one periscope
- T = total number of periscopes to be in use

The percent of items replaced, the percent replacements purchased, and the percent of items rebuilt were determined for the M44, equipment used with the M44, and for the M32 periscope. These values and the cost associated with purchase and overhaul are presented in Table 1. The actual determination of these values is presented in the appendix. These values were used for alternatives 1, 3 and 4. The replacement rate for alternative 2 is not known. It was assumed that the improved M44 would require 20 percent fewer replacements than the existing design. Comparing the M44 and the M32 replacement rates indicates this assumption provides an optimistic estimate of the PIP's effectiveness. The M32's replacement rate is 12.78 percent (i.e., the sum of total periscope replacements, head replacements and body replacement). A twenty percent reduction of M44 replacement rate results in a replacement rate of 11.58 percent. Therefore assuming a 20 percent reduction in replacement rate results in the lowest replacement rate for all alternatives considered. The yearly replacement costs and the one time implementation costs are presented in Table 2. The determination of the yearly replacement costs is presented in the appendix. The Cumulative Cost in FY77 Discounted Dollars associated with each alternative was determined for owership periods for as long as 25 years. These data are presented in Table 3 and Figure 1.

4. Comparing the costs of ownership for each alternative can be accomplished by reviewing Figure 1. The following statements can be made:

a. Alternative 1 is the least costly alternative for the first eight years of consideration, at which time alternatives 1, 2 and 3 are equal in total cost.

b. Alternative 2 breaks even in the ninth year and shows a cost savings when compared to alternative 1 in the subsequent years. However, after nine years alternative 2 shows higher costs than alternative 3, even with the optimistic assumption that alternative 2 would require 20% fewer replacements than the existing design.

c. Alternative 3 breaks even in the ninth year and in the subsequent years is the least costly when compared to the other alternatives.

d. Alternative 4 is always more costly than the other alternatives.

It should be noted that two of the assumptions made in this analysis could affect the breakeven point for alternatives 2 and 3. Firstly, it was assumed that each alternative had been implemented at the beginning of the time period. In actuality, the product improvements would have to be purchased and then phased into the systems. This will make the breakeven point greater than nine years. Secondly, it was assumed that all 1575 Sheridans would be in use during the entire period of consideration, even though only 812 were operating at the end of 1975. The projected overhaul of Sheridans could have been phased in or an estimate of the average number of Sheridans could have been used. In either case, the breakeven point would probably be extended past nine years.

Furthermore, it is important to consider the economic life of the Sheridans. If the Sheridans are going to be replaced in 5 to 10 years, then it would not be economical to implement any of the PIPs.

In addition to cost considerations, other areas that could be considered are:

a) If alternative 3 is selected, maintainability will be enhanced because of the modular construction provided by this alternative and the night vision elbow in the M551 will be interchangeable with the night vision elbows in the M60 tanks.

b) If alternative 1 or 2 is selected, difficulty may be experienced in obtaining replacement image intensifiertubes as these first generation image intensifier tubes are no longer in production.

Robert Blankert

ROBERT R. BLANKERT
Operations Research Analyst
Methodology Division

TABLE 1

Description	Percent Replaced/Yr	Percent Replacements Purchased	Percent Replacements Overhauled	Purchase Cost	Overhaul Cost
M44 Periscope	14.47	34.80	65.20	15,524	1,876
Panel FSN (1240-181-5612)	1.14	100.00	0	241	-----
Panel FSN (1240-916-5914)	7.79	100.00	0	209	-----
Cable	10.48	100.00	0	33	-----
Circuit Card	16.66	100.00	0	22	-----
M32 Periscope (total)	4.11	14.96	85.04	4,400	808
M32 Head	4.48	37.40	62.60	1,450	368
M32 Body	4.19	30.56	69.44	2,852	440

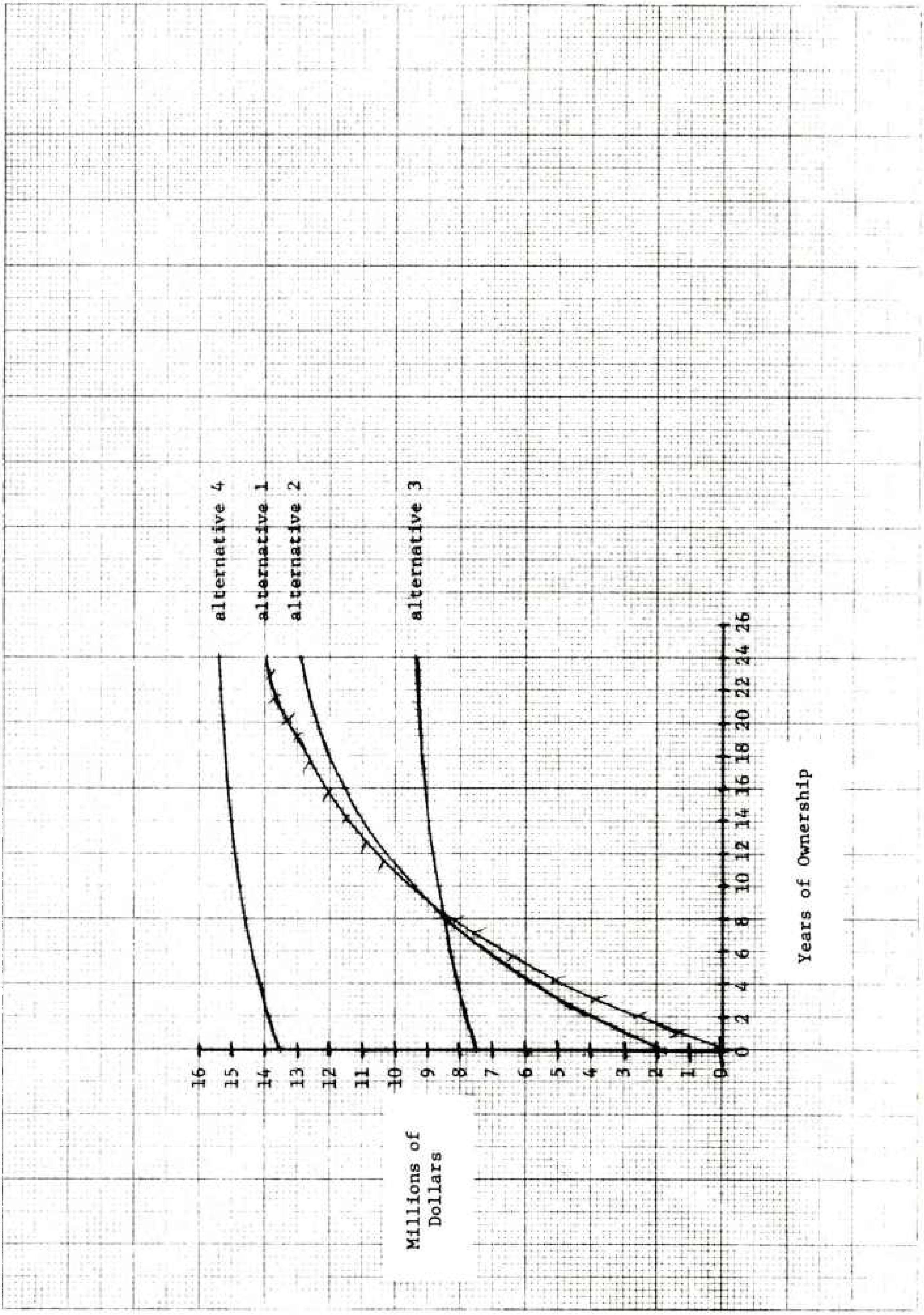
TABLE 2

Alternative	Initial PIP Cost	Yearly Maintenance Cost
1. Do nothing to existing design	0	1,551,157
2. Improve M44	$ 1,765,417	1,240,926
3. Install M32 elbow on M44	7,500,000	202,555
4. Replace M44 with M32 periscope	13,500,000	202,555

TABLE 3

CUMULATIVE COST OF IMPLEMENTING ALTERNATIVES
FY77 CONSTANT DISCOUNTED DOLLARS
ASSUMED DISCOUNT RATE 10%

Years of Ownership	Do Nothing To Existing Design	Improve M44 Periscope	Install M32 Elbow On M44 Periscope	Replace with M32 Periscope
0	0.0	$1.77M	$7.5 M	$13.5 M
1	$1.41M	2.89	7.68	13.68
2	2.69	3.92	7.85	13.85
3	3.86	4.85	8.00	14.00
4	4.91	5.70	8.14	14.14
5	5.88	6.47	8.26	14.26
6	6.75	7.17	8.38	14.38
7	7.55	7.80	8.48	14.48
8	8.27	8.38	8.58	14.58
9	8.93	8.91	8.66	14.66
10	9.53	9.39	8.74	14.74
11	10.07	9.82	8.81	14.81
12	10.56	10.22	8.88	14.88
13	11.01	10.58	8.93	14.93
14	11.42	10.90	8.99	14.99
15	11.79	11.20	9.04	15.04
16	12.13	11.47	9.08	15.08
17	12.44	11.71	9.12	15.12
18	12.72	11.94	9.16	15.16
19	12.97	12.14	9.19	15.19
20	13.20	12.32	9.22	15.22
21	13.41	12.49	9.25	15.25
22	13.60	12.64	9.28	15.28
23	13.77	12.78	9.30	15.30
24	13.93	12.91	9.32	15.32
25	14.07	13.02	9.34	15.34



Millions of Dollars
Years of Ownership
alternative 4
alternative 1
alternative 2
alternative 3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
0
2
4
6
8
10
12
14
16
18
20
22
24
26

TABLE A-1

M551 Overhaul Program

Rebuild Period	Combat Damaged Overhauls at Anniston	M551 CONUS Milage Overhauls at Anniston	M551 Europe Overhauls at Mainz
73-74	43	0	61
75-76	159	0	78
77	10	61	42
77-78 (Projected)	23	192*	56
TOTAL	335	253	237

* 50 rebuilt from Europe

TABLE A-2

Recurring Demands for
M44 Periscopes

Period	Demand
73-74	87
75-76	104

TABLE A-3

Demands for M44 Periscope
Related Equipment

Description	Non Recurring	Recurring	Purchase Cost
Panel FSN 1240-181-5612	12	3	$241
Panel FSN 1240-916-5914	96	12	209
Cable FSN 1240-906-7948	96	36	33
Circuit Card FSN 1240-946-8829	48	120	22

TABLE A-4

M44 Periscopes in Use

YEAR	1974	1975
TOTAL IN USE	974	812

TABLE A-5

Tank Overhaul Program

Period	M60	M60A1
75	376	163
76	343	74
77-7T	255	72
78	215	185
79	62	130
80	0	101
TOTALS	1251	725

TABLE A-6

Vehicles and M32 Periscopes in Use

Vehicle	Number	Periscopes
M60	3,527	7,054
M60A2	288	576
M48	963	963
TOTAL		8,593

TABLE A-7

Number of Periscopes Rebuilt and Purchased with Respective Costs

Item	Number Purchase/Yr	Purchase Cost	Number Rebuilt/Yr	Rebuilt Cost
M44	64	15,524	120	1,876
M32 Periscope	60	4,400	341	808
M32 Head	144	1,450	241	368
M32 Body	110	2,852	250	440

TABLE A-8

Recurring Demands for M32
Periscopes & Related Equipment

Complete M32 Periscope	24/Yr
M32 Head	385/Yr
M32 Body	360/Yr

1. Determination of percent of M44 periscope replaced per year.

$$P = (\overline{R} + \overline{D})/T \qquad \text{EQ 1}$$

where

P = Fraction of M44's replaced each year.

$\overline{R}$ = Average number of periscopes rebuilt with the mileage rebuild vehicles. (from Table A-1)

$\overline{D}$ = Average number of periscopes requested for field replacement per year. (from Table A-2)

T = Total number of M44's in use. (from Table A-4)

$$P = \left[\frac{253 + 237}{6} + \frac{87 + 104}{4}\right] \Big/ \left[\frac{974 + 812}{2}\right]$$

$$= .1447$$

2. Determination of fraction of M44 periscopes replacements purchased and rebuilt.

$$R_p = \frac{Np}{N_p + N_r} \qquad \text{EQ 2}$$

where

R_p = Fraction purchased

N_p = Number purchased each year from Table A-7

N_r = Number rebuilt each year from Table A-7

$$R_p = \frac{64}{120 + 64}$$

$$= .348$$

$$R_r = 1 - R_p \qquad \text{EQ 3}$$

where

R_r = Fraction rebuilt

$R_r = 1 - .348$

$= .652$

3. Now determining panel replacement rate. To get peacetime usage of panels from Table A-1:

$$\text{Faction M551 mileage overhauls} = \frac{\text{Mileage overhauls}}{\text{All overhauls}}$$

$$= \frac{253 + 237}{335 + 253 + 237}$$

$$= .6$$

Using equation 1 and data from Table A-3 to determine the fraction of panel (FSN 1240-181-5612) replaced.

$$P = \frac{(.6)(12) + 3}{893}$$

$$= .0114$$

Using a similar technique for panel (FSN 1240-916-5914)

$$P = \frac{(.6)(96) + 12}{893}$$

$$= .0779$$

4. Determining cable replacement rate using equation 1 and data from Table A-3.

$$P = \frac{(.6)(96) + 36}{893}$$

$$= .1048$$

5. Determining circuit card replacement rate using equation 4 and data from Table A-3.

$$P = \frac{(.6)(48) + 120}{893}$$

$$= .1666$$

6. Determining the yearly replacement costs for the M44 periscopes.

Cost of M44 purchased for replacement (.1447)(.348)(1575)($15,524)	=	$1,231,209
Cost of M44 rebuild for replacements (.1447)(.652)(1575)($1,876)	=	278,759
Cost of panel replacement (FSN-1240-181-5612) (.0114)(1575)($241)	=	4,327
Cost of panel replacement (FSN-1240-916-5914) (.0779)(1575)($209)	=	25,642
Cable Costs (.1048)(1575)($33)	=	5,447
Circuit card costs (.1666)(1575)($22)	=	5,773
TOTAL		$1,551,157

7. Using Equation 1 and data from Tables A-5, A-6 and A-8 to calculate the fraction of M32 periscopes replaced.

$$P = \frac{(1251 + 725)/6 + 24}{8593}$$

$$= .0411$$

8. Using equations 2 and 3 and data from Table A-7 to determine percent purchased and percent rebuilt.

$$R_p = \frac{60}{60 + 341}$$

$$= .1496$$

$$R_r = .8504$$

9. Using Equation 1 and data from Table A-7 to determine the fraction of M32 heads replaced per year.

$$P = \frac{144 + 241}{8593}$$

$$= .0448$$

10. Determining the fraction purchases & fraction rebuild using equations 2 & 3 and data from Table A-7.

$$R_p = \frac{144}{385}$$

$$= .374$$

$$R_R = 1 - .374$$

$$= .626$$

11. Using equation 1 and data from Table A-7 to determine the number of M32 bodies replaced.

$$P = \frac{110 + 250}{8593}$$

$$= .0419$$

12. Using equations 2 & 3 and data from Table A-7 to determine the fraction of M32 bodies purchased and the fraction rebuilt.

$$R_p = \frac{110}{110 + 250}$$

$$= .3056$$

$$R_R = 1 - .3056$$

$$= .6944$$

13. Determining the yearly replacement costs if the M32 periscope is used in place of the M44 periscope.

Cost of purchasing M32 periscopes (.0411)(.1496)(1575)($4400)	=	$42,610
Cost of rebuilding M32 periscopes (.0411)(.8504)(1575)($808)	=	44,479

Cost of purchasing M32 periscope heads (.0448)(.374)(1575)($1450)	=	$38,265
Cost of rebuilding M32 periscope heads (.0448)(.626)(1575)($368)	=	16,255
Cost of purchasing M32 periscope bodies (.0419)(.3056)(1575) ($2582)	=	52,072
Cost of rebuilding M32 periscope bodies (.0419)(.3056)(1575)($440)	=	8,874
Total cost of replacement on M32		$202,555

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REFERENCE OR OFFICE SYMBOL	SUBJECT
DRSAR-SAA	XM204 Howitzer Production Trade-Off Analysis

TO	FROM	DATE	CMT 1
DRSAR-AS	DRSAR-SA	25 JUL 77 Mr. Trier/plb/6370	

1. Reference is made to DF, DRSAR-AS to DRSAR-SAA and eight other Directorates, subject: 105mm Howitzer Production, XM204, dated 29 Mar 77.

2. Systems Analysis was tasked (ref 1) to provide a cost analysis on the possible procurement of XM204 Howitzers and the subsequent potential revenues received by selling overhauled M101A1 and M102 Howitzers via Foreign Military Sales (FMS). Several alternative plans were addressed which considered replacing all Army 105mm Howitzer assets with XM204 Howitzers, replacing only those 105mm Howitzers in Active Army units, or replacing either all M101A1 Howitzers or all M102 Howitzers. In addition, the FMS selling price was parameterized to show how potential revenues increase/decrease as the FMS selling price increases/decreases. Attached MFR (Incl 1) contains the results of this study.

3. Point of contact is Mr. Norman H. Trier, extension 6370.

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M. RHIAN
Director, Systems Analysis Directorate

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DRSAR-SAA

19 JUL 1977

MEMORANDUM FOR RECORD

SUBJECT: XM204 Production Trade-Off Analysis

1. Objective: Systems Analysis was tasked[1,2] to determine:

a. the cost of producing XM204 Howitzers to replace the Army's current assets of M101A1 and M102 Howitzers, and

b. the potential net revenues (total revenues minus overhaul costs) of selling overhauled M101A1 and M102 Howitzers via Foreign Military Sales (FMS).

2. Introduction: The Development Acceptance (DEVA) In-Process Review (IPR) for the XM204 Howitzer, a soft recoil 105mm Towed Howitzer, is scheduled for December 1977. Depending on the Development and Operation testing, and other considerations such as the availability of funds and other higher priority systems, the Research, Development and Acquisition Committee (RDAC) could decide to support the production of the XM204 Howitzers. If the XM204 Howitzers are produced and fielded, the current Army assets of 105mm Howitzers (Table 1)[3,4] would be replaced, and, according to the SAMPAM[5], the potential distribution of Army 105mm Howitzer assets in FY83 would be similar to that shown in Table 2. If the XM204's are fielded, there is the possibility of overhauling and selling existing Army 105mm Howitzers to FMS customers and diverting that revenue to production of XM204 Howitzers. This report addressed the potential net revenues which could be received from FMS of existing 105mm Howitzers assets and the investment which would be required to produce and field XM204 Howitzers.

3. Alternatives: Four alternatives (ALT 1, 2, 3, and 4) were identified for this analysis and are listed in Table 3. Each alternative addresses the replacement of the current Army assets of 105mm Howitzers with new production of XM204 Howitzers.

ALT 1 addresses the replacement of 105mm Howitzers in Active Army, Reserves, and National Guard units with XM204 Howitzers. This is done in accordance with potential distribution of 105mm Howitzers as found in the 3 Jan 77 SAMPAM (refer to Table 2).

ALT 2 addresses the replacement of 105mm Howitzers in Active Army only with XM204 Howitzers, while retaining a mix of M101A1 and M102 Howitzers in the Reserve and National Guard units.

TABLE 1. CURRENT ARMY ASSETS OF 105MM HOWITZERS[a,b]

	Active Army	Reserves & National Guard	Totals
M101A1	300	445	745
M102	449	135	584
Totals	749	580	1,329

[a]DRSAR-MMH DF to DRSAR-AS, subject: 105mm Howitzer Production, XM204, dated 27 Apr 77. CONFIDENTIAL

[b]Meeting between Mr. Aukland, DRSAR-MMH, and Mr. Trier, DRSAR-SA, subject: Quantity of M101A1 and M102 Howitzers in Reserves and National Guard, dated 28 Apr 77.

TABLE 2. POTENTIAL DISTRIBUTION OF ARMY ASSETS[a] OF 105MM HOWITZERS

	Active Army	Reserves & National Guard	Totals
M101A1	--	--	--
M102	248	30	278
XM204	545	573	1,118
Totals	793	603	1,396

[a]Army Materiel Plan Summary, 3 January 1977, Printout numbers G0180000M00, G0180100M00, G0180200M00, and G0180300M00. CONFIDENTIAL

TABLE 3. ALTERNATIVES

ALT 1:

a. Produce 1,118 XM204 Howitzers for Active Army, Reserves, and National Guard.

b. Overhaul 745 M101A1 Howitzers and 306 M102 Howitzers.

c. Sell, via FMS, the overhauled M101A1 and M102 Howitzers.

ALT 2:

a. Produce 793 XM204 Howitzers for Active Army only.

b. Overhaul 300 M101A1 and 426 M102 Howitzers from current Active Army assets.

c. Sell, via FMS, the overhauled M101A1 and M102 Howitzers.

ALT 3:

a. Produce 812 XM204 Howitzers for Active Army, Reserves, and National Guard.

b. Overhaul 745 M101A1 from current Army assets.

c. Sell, via FMS, the overhauled M101A1 Howitzers.

ALT 4:

a. Produce 651 XM204 Howitzers for Active Army, Reserves, and National Guard.

b. Overhaul 584 M102 Howitzers from current Army assets.

c. Sell, via FMS, the overhauled M102 Howitzers.

ALT 3 addresses the replacement of all M101A1 Howitzers in current Army assets with XM204 Howitzers, while retaining the current assets of M102 Howitzers.

ALT 4 addresses the replacement of all M102 Howitzers in current Army assets with XM204 Howitzers, while retaining the current assets of M101A1 Howitzers.

For each of the four alternatives, the potential distribution of Army 105mm Howitzers is displayed in Table 4. In addition, for each alternative, the production schedules of XM204 Howitzers, the overhaul schedules for the M101A1 and M102 Howitzers, and the FMS schedules for overhauled M101A1 and M102 Howitzers which were used in this analysis are displayed in Appendix A, Tables A-1, A-2, A-3, and A-4.

4. Approach: The analysis was consistent with guidelines in AR 11-28, Economic Analysis and Program Evaluation for Resource Management[6], in that all input data was converted to constant FY77 dollars and all monies laid out in the out years were discounted to FY77 by applying the 10% discount factors.

The investment costs of XM204 Howitzers include the Initial Production Facilities (IPF), production costs, initial provisioning and training, and publications. The per unit production cost estimate includes the hardware and support costs, test ammunition, and first destination transportation.

The per unit overhaul cost estimate for both the M101A1 and M102 Howitzers includes labor expenses, general and administration (G&A) expenses, indirect maintenance expense (IME), materiel, test ammunition, and transportation charges.

The selling price of overhauled M101A1 and M102 Howitzers for Foreign Military Sales (FMS) was calculated by applying the methodology used for the FMS of M101A1 Howitzers in 1976.[7,8] That is, the FMS selling price was determined as 80% of their respective standard prices plus the cost of overhaul and test ammunition. Then, since FMS prices are subject to change, the FMS prices were paramaterized to demonstrate how changes in selling price affect the potential revenue received.

5. Assumptions: It was assumed that:

a. The production schedule for XM204 Howitzers is feasible when considering the workload of the appropriate manufacturing facilities.

b. The XM204 Howitzer production and delivery schedules, the M101A1 and M102 Howitzer overhaul schedules, and the FMS delivery schedules shown in Tables A-1, A-2, A-3, and A-4 are valid for this analysis.

TABLE 4. POTENTIAL DISTRIBUTION OF ARMY 105MM HOWITZER ASSETS FOR EACH ALTERNATIVE

		Active Army	Reserves & National Guard	Totals
ALT 1:				
	XM204s	545	573	1,118
	M102s	248	30	278
	M101A1s	--	--	--
	Total	793	603	1,396
ALT 2:				
	XM204s	793	--	793
	M102s	--	158	603
	M101A1s	--	445	--
	Total	793	603	1,396
ALT 3:				
	XM204s	344	468	812
	M102s	449	135	584
	M101A1s	--	--	--
	Total	793	603	1,396
ALT 4:				
	XM204s	493	158	651
	M102s	--	--	--
	M101A1s	300	445	745
	Total	793	603	1,396

c. For FMS, the M101A1 and M102 Howitzers will be delivered the same year in which they are overhauled.

d. Cost of the test ammunition for the overhauled M101A1 and M102 Howitzers is equal to the cost of test ammunition for the XM204 Howitzer.

e. The per unit production cost for the XM204 Howitzers is valid for production of 1,118 howitzers for ALT 1, as well as for production of lesser quantities of howitzers for ALT 2, 3 and 4.

f. The cost for IPF, initial provisioning and training, and publications for the XM204 Howitzers are applied equally to all four alternatives.

g. Transportation costs, estimated as 3% of the Hardware and Support (Engineering and Production) Costs of the new production of XM204 Howitzers, are applied equally to the M101A1, M102, and XM204 Howitzers.

h. Transportation costs for FMS will be paid by the foreign country and are, therefore, not included in the estimation of net revenues received from FMS of M101A1 and M102 Howitzers.

6. Data: The following data were used in this analysis:

a. Current Army assets of 105mm Howitzers (Table 1) are 745 M101A1 and 584 M102 Howitzers.

b. XM204 Howitzer production and delivery schedules, M101A1 and M102 Howitzer overhaul schedules, and FMS schedules for FY81 through FY87 (Tables A-1, A-2, A-3 and A-4).

c. FMS selling prices for overhauled M101A1 and M102 Howitzers (Table A-5) are $31,193 and $126,040, respectively.

d. Overhaul costs for M101A1 and M102 Howitzers[9] (Table A-6) are $12,820 and $36,760, respectively.

e. XM204 Howitzer estimates are $121.5K for new production costs per unit, $6.29M for Initial Production Facilities (IPF), $2.9M for initial provisioning and training, and $3.4M for publications (Table A-7).

f. Estimated transportation cost is $3.6K for the XM204 Howitzer, the M101A1 Howitzer, and the M102 Howitzer.

7. Results: Table 5 displays the investment costs that would be incurred to procure and field XM204 Howitzers for each alternative. Production costs were calculated each year according to the number of howitzers produced (refer to production schedules in Tables A-1, A-2, A-3, and A-4). All costs were then discounted to constant FY77 dollars.

The costs of overhauling the M101A1 and M102 Howitzers in preparation for FMS customers are displayed in Table A-8.These overhaul costs would be paid out of the revenues obtained from the FMS customers. Figures 1, 2, 3 and 4 display the potential net revenues (total revenues (Figures A-1,A-2, A-3 and A-4) minus overhaul costs (Table A-9)) which could be received from FMS of M101A1 and M102 Howitzers for ALT 1, 2, 3 and 4, respectively. Each figure displays two curves (straight lines); one shows the potential net revenues obtained by selling M101A1 Howitzers, and the other shows the potential revenues obtained by selling M102 Howitzers.

In order to exemplify the use of the figures, two cases shall be discussed. In the two cases, different FMS selling prices will be used to show how revenue varies with selling prices. The selling prices used are:

	M101A1	M102
Case 1	∿$31K	∿$126K
Case 2	∿$97K	∿$ 97K

The values for Case 1 were determined as 80% of their respective standard prices plus the cost of overhaul and test ammunition (see Table A-5). The values for Case 2 were determined as 80% of new production costs of XM204 Howitzers. Net FMS revenues corresponding to the selling prices were obtained from Figures 1, 2, 3 and 4 and are displayed in Table 6.

TABLE 6. NET FMS REVENUE ($M̄)
(In Constant Discounted FY77 Dollars)

	Quantity sold		Revenues for Case 1	Revenues for Case 2
	M101A1	M102	M101A1 + M102 = TOTAL	M101A1 + M102 = TOTAL
ALT 1	745	306	4.8 + 13.0 = $17.8M̄	28.5 + 8.6 = $37.1M̄
ALT 2	300	426	2.0 + 18.6 = $20.6M̄	12.2 + 12.2 = $24.4M̄
ALT 3	745	-	5.1 + 0.0 = $ 5.1M̄	30.4 + 0.0 = $30.4M̄
ALT 4	-	586	0.0 + 26.0 = $26.0M̄	0.0 + 17.1 = $17.1M̄

TABLE 5. XM204 HOWITZER FIELDING COSTS
(In Constant Discounted FY77 Dollars)

ALT 1	
XM204 Production (Qty = 1,118)	$71.9M
Initial Production Facilities	4.6M
Initial Provisioning and Training	2.2M
Publications	2.4M
TOTAL	$81.1M
ALT 2	
XM204 Production (Qty = 793)	$53.7M
Initial Production Facilities	4.6M
Initial Provisioning and Training	2.2M
Publications	2.4M
TOTAL	$62.9M
ALT 3	
XM204 Production (Qty = 812)	$54.8M
Initial Production Facilities	4.6M
Initial Provisioning and Training	2.2M
Publications	2.4M
TOTAL	$64.0M
ALT 4	
XM204 Production (Qty = 651)	$45.0M
Initial Production Facilities	4.6M
Initial Provisioning and Training	2.2M
Publications	2.4M
TOTAL	$54.2M

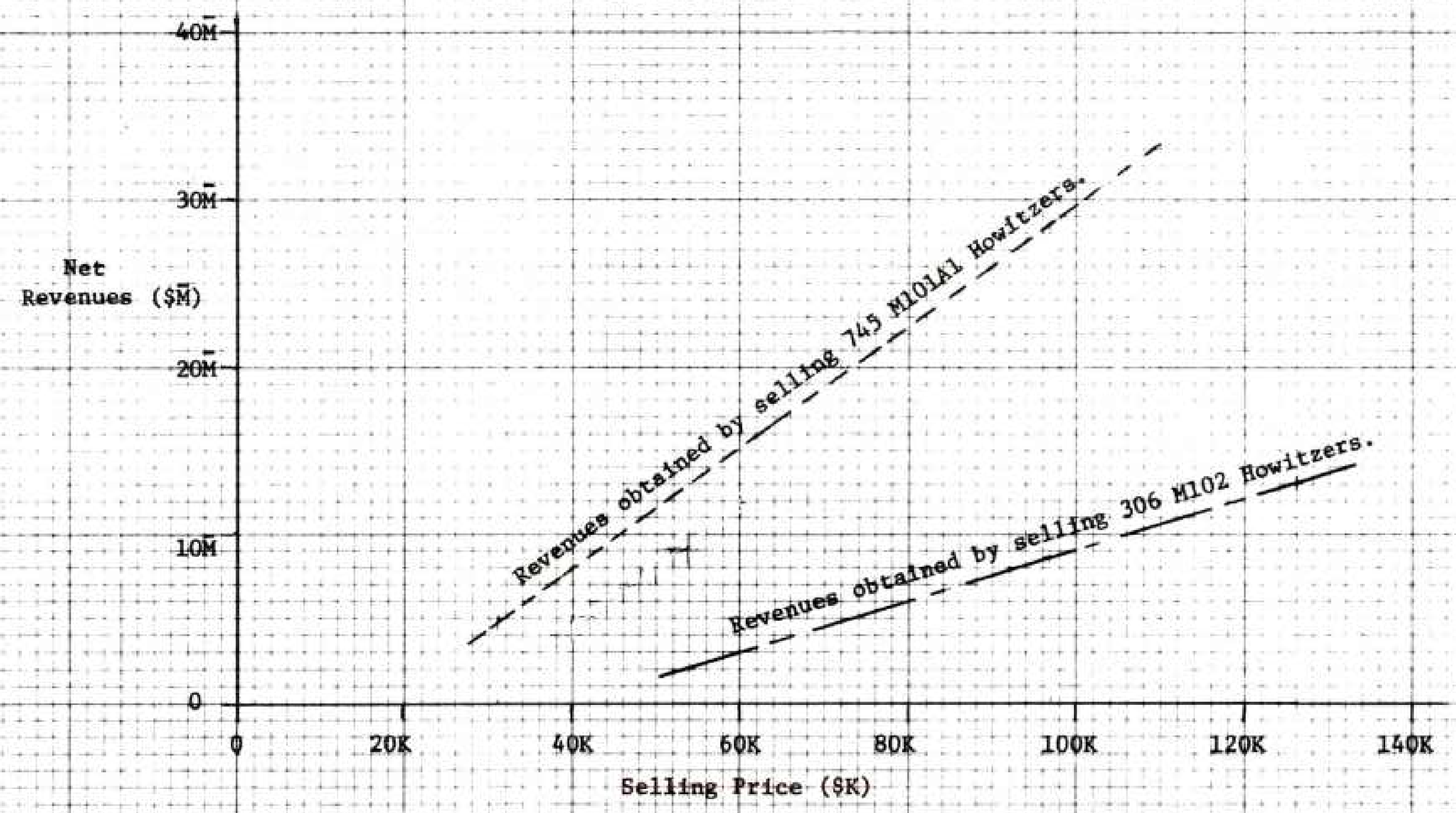

Figure 1. Potential Net Revenues From FMS for ALT 1
(In Constant Discounted FY77 Dollars)

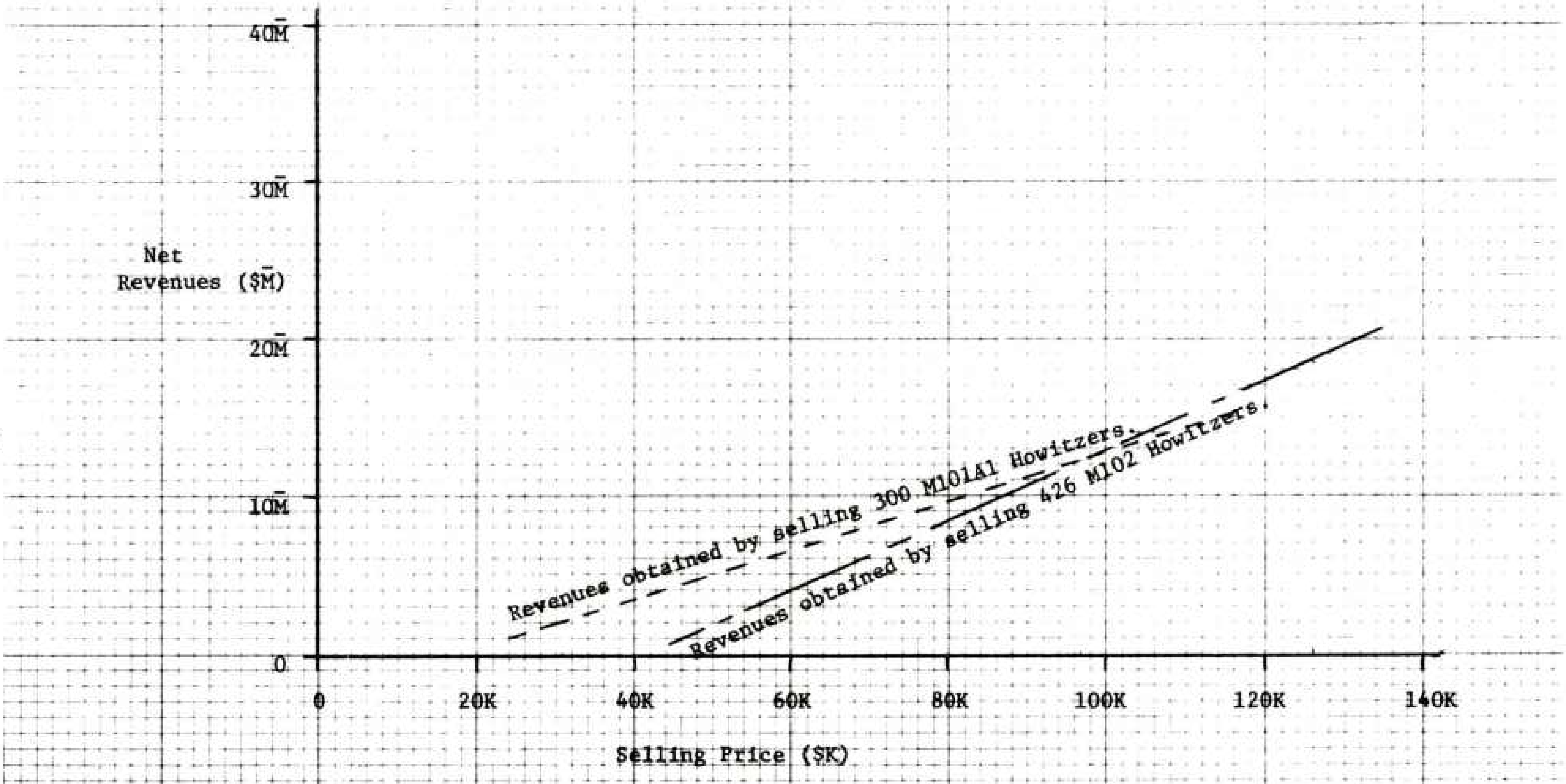

Figure 2. Potential Net Revenues from FMS for ALT 2.
(In Constant Discounted FY77 Dollars)

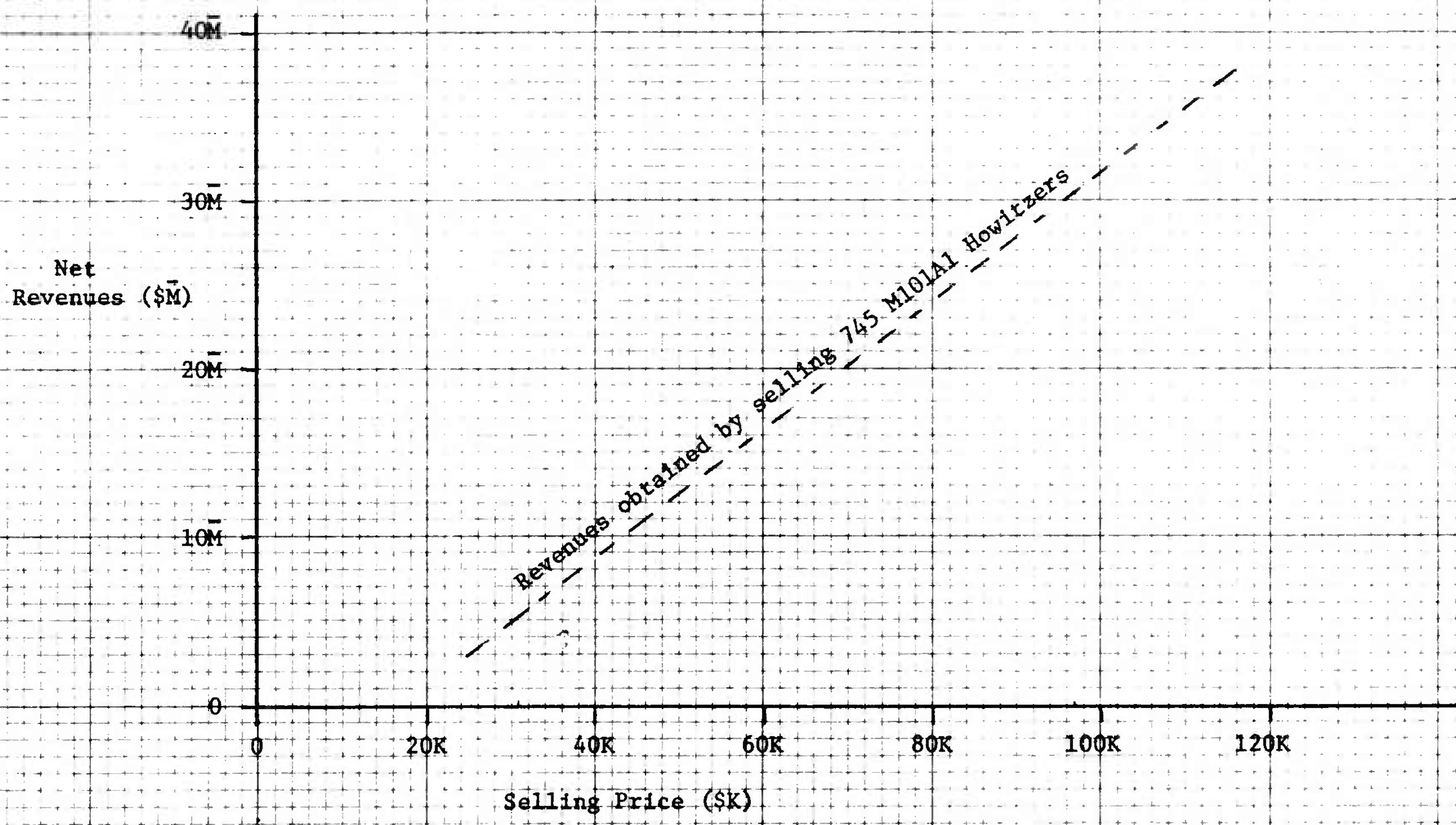

Figure 3. Potential Net Revenues From FMS for ALT 3
(In Constant Discounted FY77 Dollars)

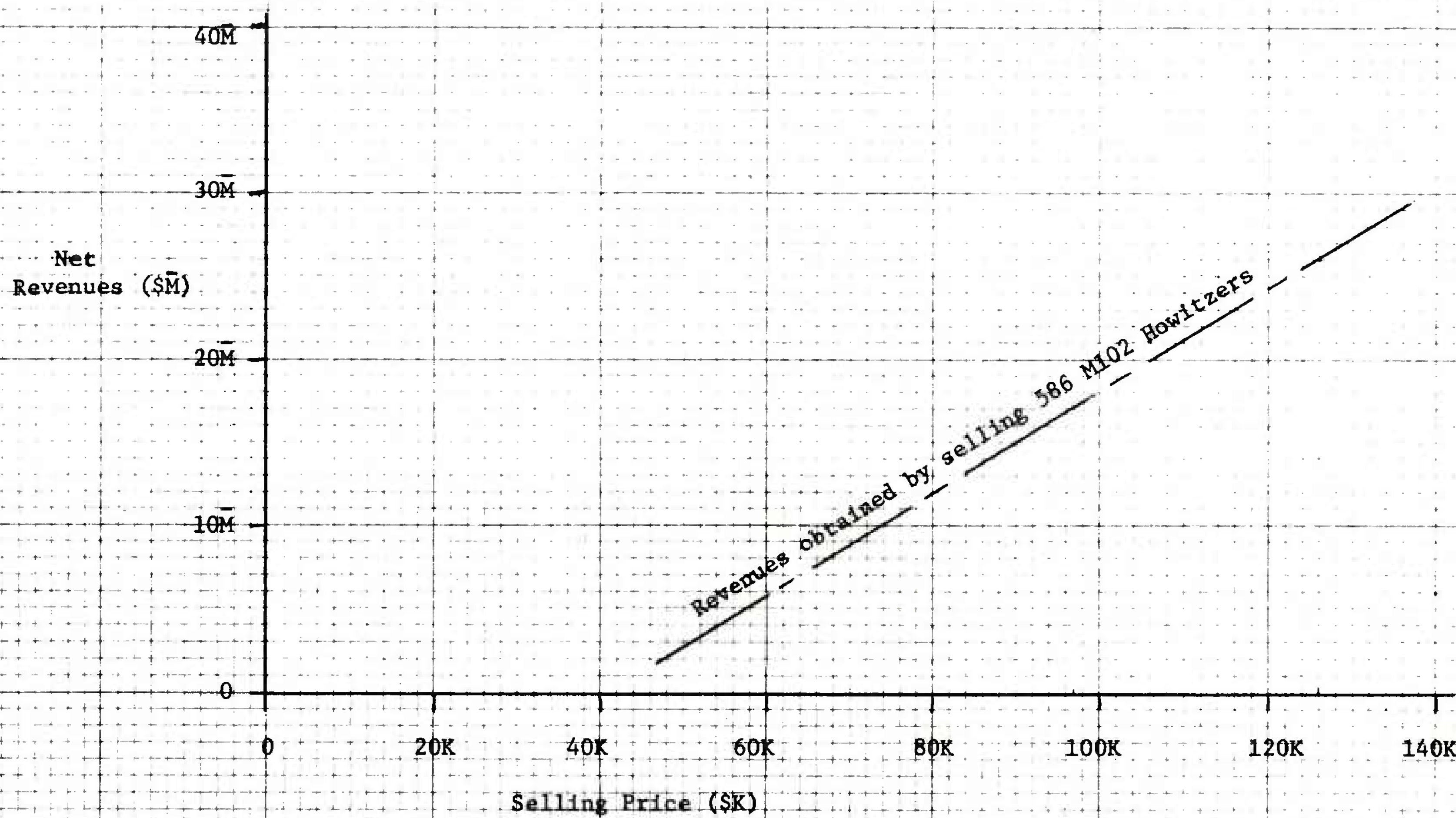

Figure 4. Potential Net Revenues from FMS for ALT 4
(In Constant Discounted FY77 Dollars)

The net revenues shown in Table 6 would probably go to the US Government General Revenue Fund for subsequent Congressional appropriation. If these funds were reverted to the XM204 program, the net costs of each alternative could be calculated as is done in Table 7.

TABLE 7. ALTERNATIVE NET COSTS FOR XM204 HOWITZERS
(In Constant Discounted FY77 Dollars)

Case 1:	ALT 1	ALT 2	ALT 3	ALT 4
XM204 Fielding Costs	$81.1M	$62.9M	$64.0M	$52.2M
Net FMS Revenues	17.8	20.6	5.1	26.0
Net Costs	$63.3M	$42.3M	$58.9M	$26.2M

Case 2:	ALT 1	ALT 2	ALT 3	ALT 4
XM204 Fielding Costs	$81.1M	$62.9M	$64.0M	$52.2M
Net FMS Revenues	37.1	24.4	30.4	17.1
Net Costs	$44.0M	$38.5M	$33.6M	$35.1M

8. Summary: For each alternative plan of fielding XM204 Howitzers, the discounted investment costs were determined. These costs included Initial Production Facility (IPF) costs, production and transportation, initial provisioning and training, and publications. It was determined that:

a. For ALT 1, if 1,118 XM204 Howitzers were produced and delivered to Active Army, Reserve, and National Guard units, an investment of $81M will be required.

b. For ALT 2, if 793 XM204 Howitzers were produced and delivered to Active Army units only, an investment of $63M would be required.

c. For ALT 3, if 812 XM204 Howitzers are produced and delivered to Active Army, Reserve, and National Guard units to replace all M101A1 Howitzers, an investment of $64M would be required.

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d. For ALT 4, if 651 XM204 Howitzers were produced and delivered to Active Army, Reserve, and National Guard units to replace all M102 Howitzers, an investment of $54M would be required.

For each of the four alternative plans addressed in this analysis, potential net revenues (total revenues minus overhaul costs) were calculated for the sale of overhauled M101A1 and M102 Howitzers to Foreign Military Sales (FMS) customers. Figures 1, 2, 3 and 4 display those net revenues for ALT 1, 2, 3 and 4, respectively.

According to current policy, the FMS selling price for overhauled M101A1 and M102 Howitzers would be ∿$31K and ∿$126K, respectively (Table A-5). At these selling prices, the potential net revenues which could be received for ALT 1, 2, 3 and 4 are $18M, $21M, $5M, and $26M respectively. It should be noted that potential revenues depend directly on the quantities of howitzers sold. For example, if M102 Howitzers could not be sold (say, for example, due to too high a price or low desirability), and only M101A1 Howitzers were sold, the potential net revenues for ALT 1, 2, 3 and 4 would be only $5M, $2M, $5M, and $0M respectively. Lastly, if different selling prices could be justified, the subsequent potential net revenues could be obtained from Figures 1, 2, 3 and 4 as demonstrated in this report.

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REFERENCES

1. DRDAR-XM DF to DRSAR-AS, subject: XM204 Review Project, dated 23 Mar 77.

2. DRSAR-AS DF to 9 Directorates, subject: 105mm Howitzer Production, dated 29 Mar 77.

3. DRSAR-MMH DF to DRSAR-AS, subject: 105mm Howitzer Production, XM204 (U), dated 27 Apr 77, classified CONFIDENTIAL.

4. Meeting between Mr. Aukland, DRSAR-MMH, and Mr. Trier, DRSAR-SA, subject: Quantity of M101A1 and M102 Howitzers in Reserves and National Guard, dated 28 Apr 77.

5. Army Materiel Plan Summary, 3 January 1977, Printout numbers G0180000M00, G0180100M00, G0180200M00, and G0180300M00. CONFIDENTIAL.

6. AR 11-28, Economic Analysis and Program Evaluation for Resource Management, dated 15 Jan 76.

7. FONECON, Mr. Aukland, DRSAR-MMH and Mr. Trier, DRSAR-SA, subject: Standard Price of M101A1 and M102 Howitzers and Determination of FMS Selling Price, dated 25 Apr 77.

8. DALO-ILP Message, 132055Z Sep 76. CONFIDENTIAL.

9. DRSAR-CPE DF to DRSAR-ASA, subject: 105mm Howitzer Production, XM204, dated 12 Apr 77.

TABLE A-1. SCHEDULES FOR ALTERNATIVE 1

	FY81	FY82	FY83	FY84	FY85	FY86	FY87	TOTAL
<u>ALT 1</u>:								
XM204 production schedule[a]	8	50	168	288	288	316	--	1,118
delivery sch.: Active Army[b]	-	33	109	228	175	--	--	545
Res. & Nat. Guard[b]	-	--	--	--	113	302	158	573
Overhaul sch.: Act. Army[c]	-	30	100	210	161	--	--	501
Res. & N.G.[c]	-	--	--	--	108	290	152	550
M101A1: from Act. Army[d]	-	18	60	126	96	--	--	300
Res. & N.G.[d]	-	--	--	--	87	235	123	445
M102 : from Act. Army[d]	-	12	40	84	65	--	--	201
Res. & N.G.[d]	-	--	--	--	21	55	29	105
FMS Delivery sch.: M101A1[e]	-	18	60	126	183	235	123	745
M102[e]	-	12	40	84	86	55	29	306

[a]Obtained from MAJ Roddy, DRDAR-XM (RIA).

[b]Number of Howitzers delivered in a given year equals 1/2 of the number of XM204 Howitzers produced the prior year plus 1/2 of the number of XM204 Howitzers produced that same year. E.g., in FY83, the number of Howitzers delivered = 1/2(50) + 1/2(168) = 109.

[c]The overhaul schedule is calculated by multiplying the ratio of the number of M101A1 and M102 Howitzers replaced and the number of XM204 Howitzers fielded times the delivery schedule. E.g., for FY83, the number of Howitzers overhauled from the Active Army equals (501 ÷ 545) x 109 = 100.

[d]M101A1 Howitzers and M102 Howitzers are overhauled concurrently according to the ratio in which they are replaced in Active Army, and then in the Reserves and National Guard. E.g., the number of M101A1 Howitzers overhauled in FY83 equals (300 ÷ 501) x 100 = 60: the number of M102 Howitzers equals (201 ÷ 501) x 100 = 40.

[e]For FMS, the M101A1 and M102 Howitzers will be delivered the same year in which they were overhauled.

TABLE A-2. SCHEDULES FOR ALTERNATIVE 2

	FY81	FY82	FY83	FY84	FY85	FY86	FY87	TOTAL
ALT 2:								
XM204 Production Schedule	8	50	168	288	279	--	--	793
Delivery Schedule[a]	-	33	109	228	284	139	--	793
Overhaul Sch.: from Act. Army[b]	-	30	100	209	260	127	--	726
M101A1[c]	-	12	41	86	108	53	--	300
M102[c]	-	18	59	123	152	74	--	426
FMS Delivery Sch.: M101A1[d]	-	12	41	86	108	53	--	300
M102[d]	-	18	59	123	152	74	--	426

[a]Number of Howitzers delivered in a given year equals 1/2 of the number of XM204 Howitzers produced the prior year plus 1/2 of the number of XM204 Howitzers produced that same year. E.g., in FY83, the number of Howitzers delivered equals 1/2(50) + 1/2(168) = 109.

[b]The overhaul schedule is calculated by multiplying the ratio of the number of M101A1 and M102 Howitzers replaced and the number of XM204 Howitzers fielded times the delivery schedule. E.g., for FY83, the number of Howitzers overhauled equals (726 ÷ 793) x 109 = 100.

[c]M101A1 and M102 Howitzers are overhauled concurrently according to the ratio in which they are replaced. E.g., the number of M101A1 Howitzers overhauled in FY83 equals (300 ÷ 726) x 100 = 41: the number of M102 Howitzers equals (426 ÷ 726) x 100 = 59.

[d]For FMS, the M101A1 and M102 Howitzers will be delivered the same year in which they were overhauled.

TABLE A-3. SCHEDULES FOR ALTERNATIVE 3

	FY81	FY82	FY83	FY84	FY85	FY86	FY87	TOTAL
ALT 3:								
XM204 Production Schedule	8	50	168	288	288	10	--	812
Delivery Sch.: Active Army[a]	-	33	109	202	--	--	--	344
Res. & N.G.[a]	-	--	--	26	288	154	--	468
Overhaul Sch.: from Act. Army[b]	-	29	95	176	--	--	--	300
Res. & N.G.[b]	-	--	--	25	274	146	--	445
FMS Delivery Sch: M101A1[c]	-	29	95	201	274	146	--	745

[a]Number of Howitzers delivered in a given year equals 1/2 of the number of XM204 Howitzers produced the prior year plus 1/2 of the number of XM204 Howitzers produced that same year. E.g., in FY83, the number of Howitzers delivered equals $1/2(50) + 1/2(168) = 109$.

[b]The overhaul schedule is calculated by multiplying the ratio of the number of M101A1 and M102 Howitzers replaced and the number of XM204 Howitzers fielded times the delivery schedule. E.g., for FY83, the number of Howitzers overhauled equals $(300 \div 344) \times 109 = 95$.

[c]For FMS, the M101A1 and M102 Howitzers will be delivered the same year in which they were overhauled.

TABLE A-4. SCHEDULES FOR ALTERNATIVE 4

	FY81	FY82	FY83	FY84	FY85	FY86	FY87	TOTAL
ALT 4:								
XM204 Production Schedule	8	50	168	288	137	--	--	651
Delivery Sch.: Active Army[a]	-	33	109	228	123	--	--	493
Res. & N.G.[a]	-	--	--	--	90	68	--	158
Overhaul Sch.: from Act. Army[b]	-	30	99	208	112	--	--	449
Res. & N.G.[b]	-	--	--	--	77	58	--	135
FMS Delivery Sch: M102[c]	-	30	99	208	189	58	--	584

[a]Number of Howitzers delivered in a given year equals 1/2 of the number of XM204 Howitzers produced the prior year plus 1/2 of the number of XM204 Howitzers produced that same year. E.g., in FY83, the number of Howitzers delivered equals 1/2(50) + 1/2(168) = 109.

[b]The overhaul schedule is calculated by multiplying the ratio of the number of M101A1 and M102 Howitzers replaced and the number of XM204 Howitzers fielded times the delivery schedule. E.g., for FY83, the number of Howitzers overhauled equals (449 ÷ 493) x 109 = 99.

[c]For FMS, the M101A1 and M102 Howitzers will be delivered the same year in which they were overhauled.

TABLE A-5. ESTIMATED FMS SELLING PRICE[a,b]

M101A1 Howitzer:

80% of the standard price of $21,254	=	$17,003
Overhaul cost[c]	=	12,820
Test ammunition[c]	=	1,370
FMS Selling Price	=	$31,193

M102 Howitzer:

80% of the standard price of $109,887	=	$ 87,910
Overhaul cost[c]	=	36,760
Test ammunition[c]	=	1,370
FMS Selling Price	=	$126,040

[a]Method to compute FMS selling price and standard prices for the M101A1 and M102 Howitzers were obtained via FONECON between Mr. Aukland, DRSAR-MM and Mr. Trier, DRSAR-SA on 25 April 1977.

[b]DALO-ILP MSG 132055z Sep 76 (C).

[c]Data obtained from DRSAR-CPE DF to DRSAR-ASA, subject: 105mm Howitzer Production, XM204, dated 12 April 1977. Cost of test ammunition for the M101A1 and M102 Howitzers was assumed equal to the cost of test ammunition that was estimated for the XM204 Howitzer.

TABLE A-6. OVERHAUL COSTS ESTIMATES FOR M101A1 AND M102 HOWITZERS[a]

(In Constant FY77 Dollars)

M101A1 OVERHAUL - Pron M17 OE 302021OH3

Labor	$ 2,894.52
General & Administrative (G&A)	709.81
Indirect Maintenance Expense (IME)	2,447.98
Materiel	6,767.98
TOTAL	$12,820.29

M102 OVERHAUL - Pron M16 DF 301091OH3

Labor	$ 3,025.10
G&A	544.77
IME	2,834.59
Materiel	27,791.13
FY76 TOTAL	$34,195.59
	x 1.0750
FY77 TOTAL	$36,760.20

[a]DRSAR-CPE DF to DRSAR-ASA, subject; 105mm Howitzer Production, XM204, dated 12 Apr 77.

TABLE A-7. UNIT AND PROGRAM COSTS OF THE XM204 HOWITZER[a]
(In Constant FY77 Dollars)

<u>Per Unit Cost of XM204 Howitzer Production</u>

Hardware and Support	$120,123.20
Test Ammunition	1,370.35
	$121,493.65

	FY79	FY80	FY81
Initial Production Facilities	--	$1.0M	$5.29M
Initial Provisioning	$0.4M	$0.9M	$1.6M
Publications	--	--	$3.4M

[a]DRSAR-CPE DF to DRSAR-ASA, subject: 105mm Howitzer Production, XM204, dated 12 Apr 77.

TABLE A-8. M101A1 and M102 HOWITZER OVERHAUL COSTS
(In Constant Discounted FY77 Dollars)

	M101A1		M102	
	Qty	Cost	Qty	Cost
ALT 1	745	$6.4M	306	$ 6.5M
ALT 2	300	2.8M	426	9.2M
ALT 3	745	6.8M	-	-
ALT 4	-	-	586	12.9M

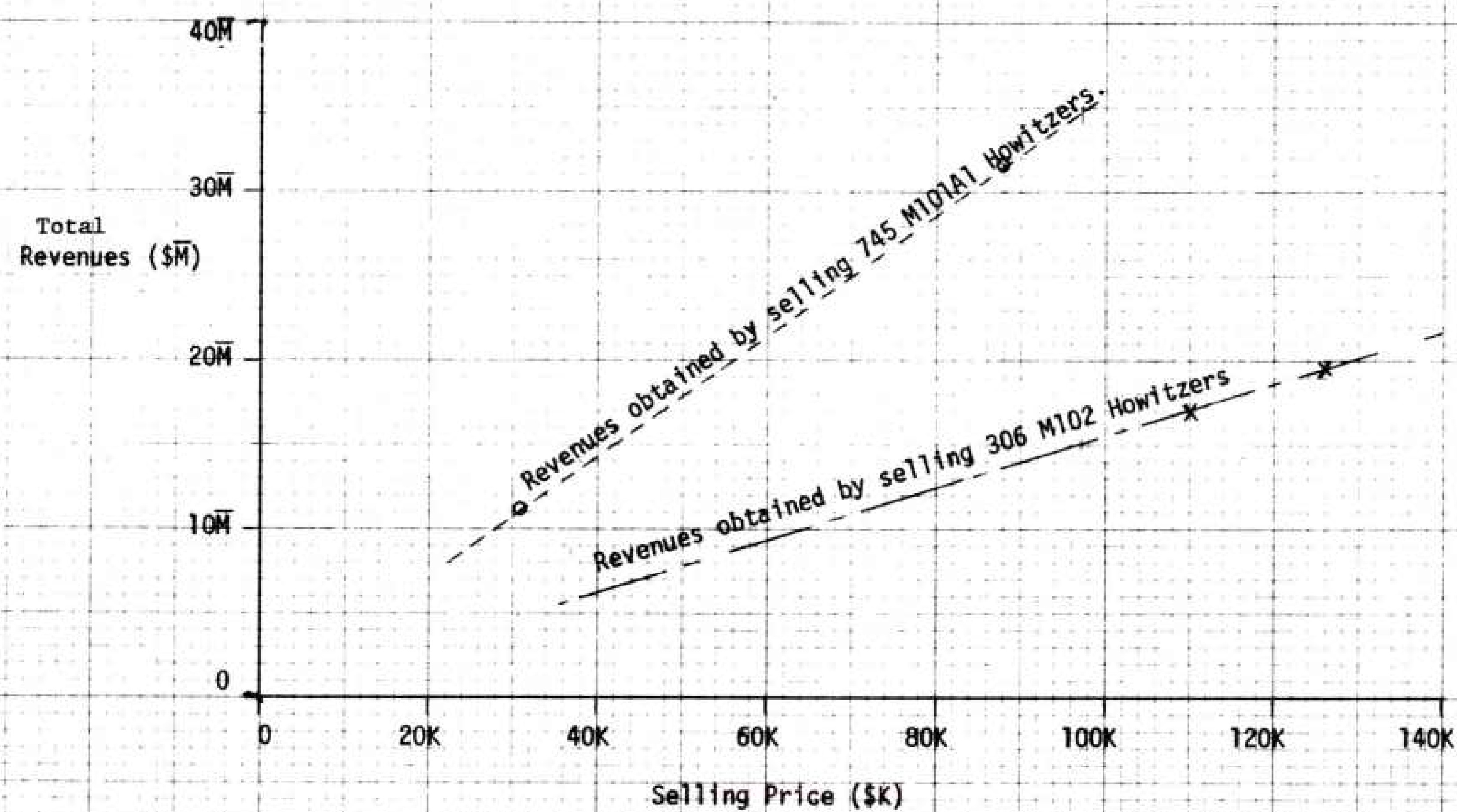

Figure A-1. Potential Revenues From FMS For ALT 1
(IN Constant Discounted FY77 Dollars)

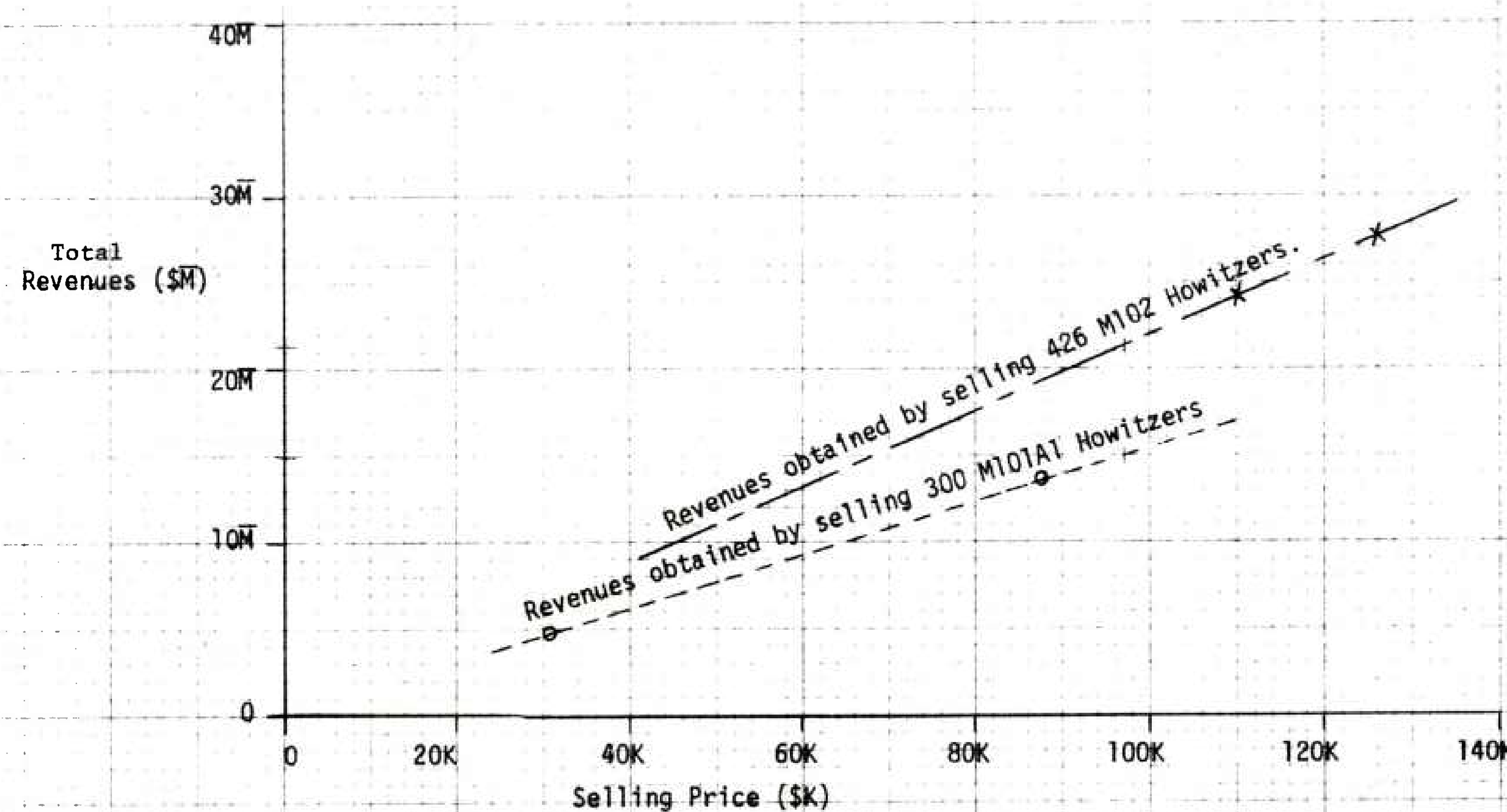

Figure A-2. Potential Revenues From FMS For ALT 2 (In Constant Discounted FY77 Dollars)

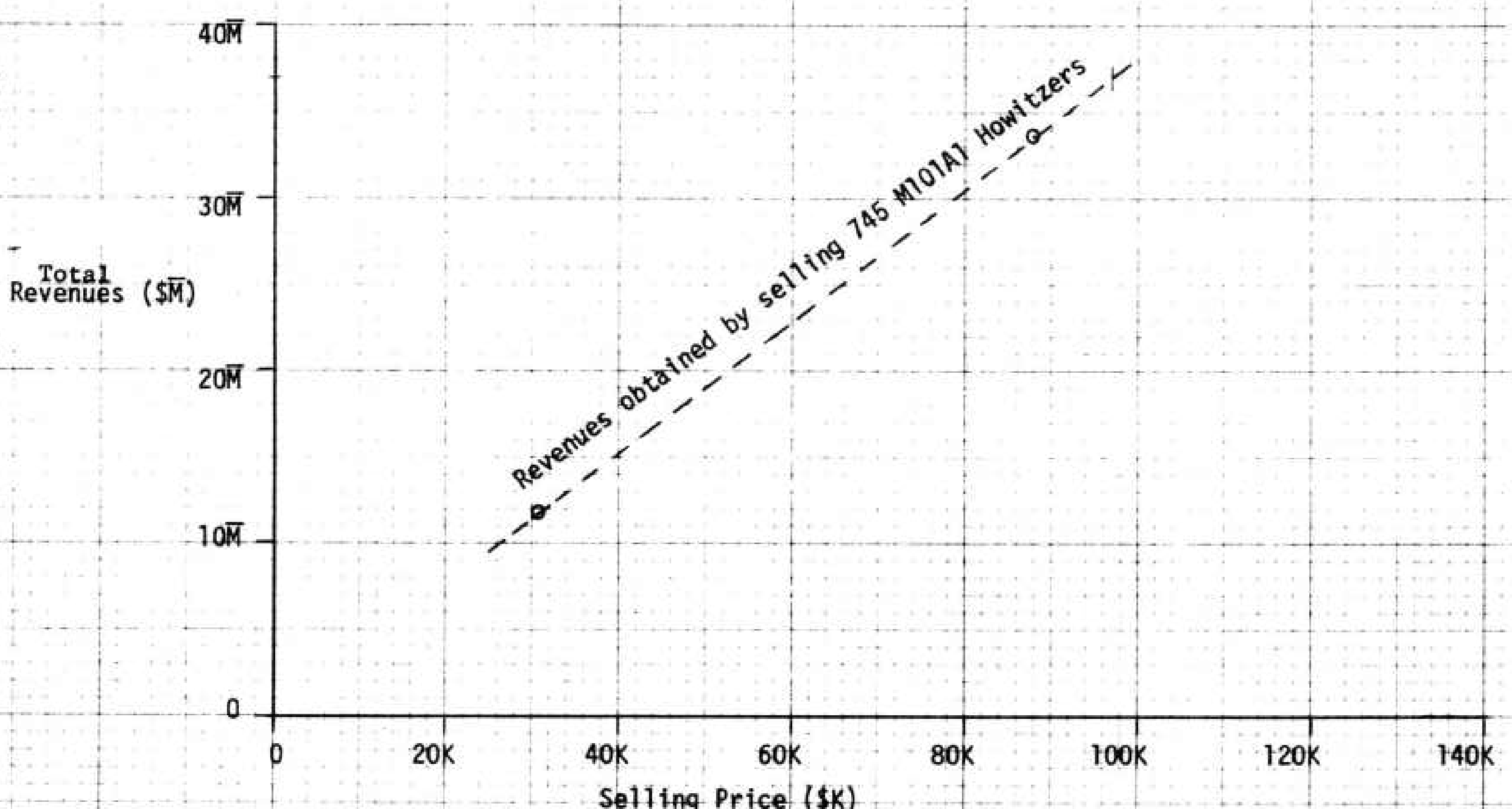

Figure A-3. Potential Revenues From FMS For ALT 3

(In Constant Discounted FY77 Dollars)

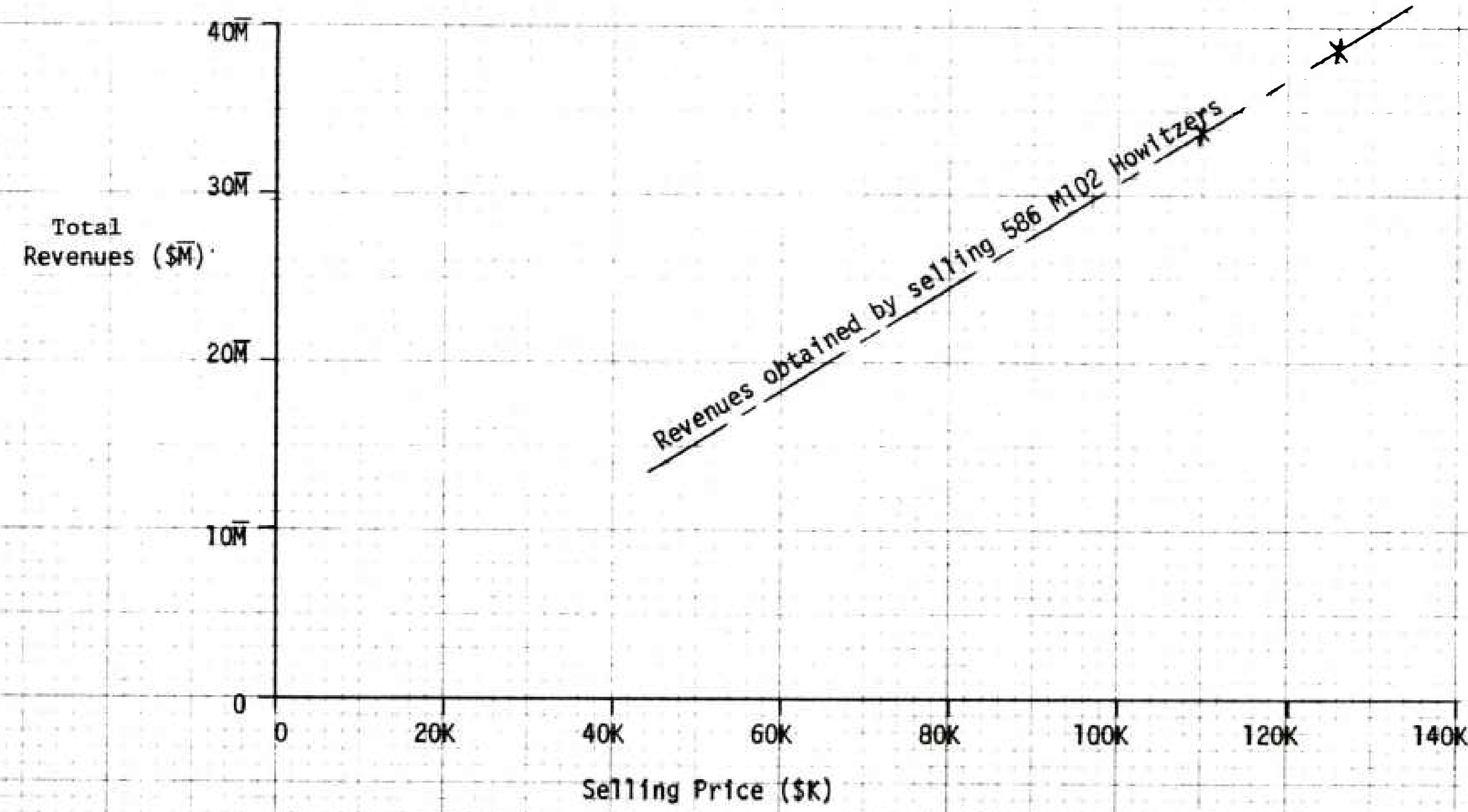

Figure A-4. Potential Revenues From FMS For ALT 4

(In Constant Discounted FY77 Dollars)

DEPARTMENT OF THE ARMY
HEADQUARTERS, UNITED STATES ARMY ARMAMENT MATERIEL READINESS COMMAND
ROCK ISLAND, ILLINOIS 61201

REPLY TO
ATTENTION OF:

DRSAR-SAM

1 AUG 1977

SUBJECT: Analysis of Interior Ballistic Safety Test Data from the M110A1 Using the M188E1 Propelling Charge Subjected to Several Treatments

Project Manager
Cannon Artillery Weapons Systems
ATTN: DRCPM-M110E2 (Mr. B. Walters)
Rock Island, IL 61201

1. References:

a. Conversations between Mr. B. Walters (DRCPM-M110E2) and Mr. G. Schlenker (DRSAR-SAM), during Jun 77, subject: firing safety of the M188E1 propelling charge.

b. MFR, DRSAR-SAM, 29 Jul 77, subject as above (Incl 1).

2. Pursuant to your request (Ref a) to DRSAR-SA to investigate various aspects of a potential firing safety problem with the M188E1 propelling charge, this office has completed an analysis of the subject data and has formed certain conclusions. Details concerning the methods of analysis and findings are contained in the attached memorandum (Ref b). These conclusions have implications for technical management of the M188E1 charge to include product improvements and are given below.

3. The principal finding is that a significant statistical difference exists between the interior ballistics of sequentially rough handled charges (set A) and those which are not, i.e., all others (set B). A difference exists in the functional relationship between maximum chamber pressure, p_{max}, and chamber pressure differential, Δp. An additional difference was discovered in the probability distribution of the random variable Δp. Together these differences produce quite different risks of a catastrophic failure for sets A and B.

4. Based upon a goal for the failure rate of 10^{-6}, it is estimated that the rate for sequentially rough handled charges (set A) exceeds this goal by a factor of 64, at low temperatures and by a factor of 22 at high temperatures, whereas charges in set B meet this goal. It is inferred that certain physical changes, possibly defects, are produced in the M188E1 charge due to rough handling.

5. It is fair to assert that if the sequential rough handling procedure is representative of the treatment received in typical transportation and use, that this charge would fail to achieve its safety goal. In the light of the differences between the performance of charges in sets A and B and of defects discovered in a breakdown of rough-handled charges, it is recommended that the PM110E2 pursue the goal of identifying and remedying the physical causes for these differences.

6. Clearly, a product improvement must be initiated if the risk of a catastrophic failure is to be reduced to 10^{-6}. Until this improvement is accomplished and demonstrated, it is prudent to restrict the firing of M188E1 charges to those which demonstrably have not been rough handled. It is also recommended that a firing test program (to be defined) be conducted on future product-improved, unmodified charges which have been sequentially rough handled to assess the risk of failure in this charge. Inferences with respect to firing safety of a particular charge design drawn from a different charge are not warranted.

M. Rhian

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as

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Director
Systems Analysis Directorate

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DRSAR-EN, ATTN: Dr. Ambrosini w incl

DRSAR-SAM

29 JUL 1977

MEMORANDUM FOR RECORD

SUBJECT: Analysis of Interior Ballistic Safety Test Data from the M110A1 Using the M188E1 Propelling Charge Subjected to Several Treatments

1. References:

a. Informal Test Data Sheets, acquired from DRCPM-M110E2, subject: Maximum Pressure Data DT II Test of the 8-Inch Howitzer, M188E1 Charge (Safety Phase).

b. MFR, STEAP-MT-G, 11 Apr 77, subject: M188E1 Charge--Max Negative ΔP Results.

c. MFR, DRSAR-SAM, 23 Jun 77, subject: Statistical Methods Pertinent to a Potential Ignition Problem in the M188E1 Propelling Charge.

d. Extracts from a Presentation on Charge Safety by BRL Representatives: I. May, et.al., acquired from DRCPM-M110E2.

2. Background

The author was asked by the PM-110E2 to review a potential safety problem in the M110A1 SP howitzer when using the M188E1 propelling charge. A catastrophic failure of this system can occur if the maximum chamber pressure, p_{max}, exceeds a critical value, beyond which the base of projectile fails causing combustion gases to enter the warhead cavity. Of course, this potential problem is not unique to the M110 system; however, the quantification of risk of failure and causes of failure may be peculiar to this system. The latter details, discussed in this memorandum, are derived from the safety test data of Ref a.

3. During ignition and before shot start, the pressure waves within the combustion chamber produce a transient pressure difference between the

Incl 1

front and rear of the chamber. The maximum algebraic value of the front minus rear pressure differential, Δp, occuring during ignition has been found to correlate with p_{max}. Although random variation of p_{max} from round to round occurs at a constant value (level) of Δp, additional variation of p_{max} occurs due to the stochastic nature of Δp and the functional dependence of p_{max} on Δp.

4. The data of Ref a had previously been analyzed by STEAP-MT-G (Ref b) with the intention of discovering the relationship(s) between p_{max} and Δp and identifying the probability distribution function of the random variable Δp. Several candidate types of cumulative distribution functions (c.d.f.) on Δp were examined. Parameters for each type of c.d.f. were estimated from the data. Goodness-of-fit tests (Chi-squared and Kolmogorov-Smirnov) were applied; and, on the basis of minimum risk of error, it was concluded that the two-parameter Weibull distribution best fits the Δp data. The present analysis supports this conclusion. Within its scope the previous analysis appears to be well done. Additionally, Ref b indicated that further analysis would be conducted pending the acquisition of more firing data. It is noted that the only division or segregation of the data for analysis purposes made in Ref b was the separate analysis of cold (-50 deg F) and hot (145 deg F) temperature-conditioned charges. Although a variety of charge preconditions exist--secured cargo, temperature soak, sequential rough handling, and no special treatment--these data were pooled to assess the c.d.f. of Δp. The data from each of these charge treatments were treated discretely in our analysis, and a variety of powerful statistical tests applied.

5. One of the points made in this memorandum is that the aggregation of data from all charge treatments obscures important differences in the effect of treatment. Although it is not obvious from an initial

inspection of the data, it will be shown that the type of treatment (charge precondition) affects not only the c.d.f. of Δp but the functional relation: $p_{max}(\Delta p)$. Since these relationships are essential ingredients to a risk analysis of the M110A1-M188E1 system, the risk of a catastrophic failure will be shown to depend upon charge treatment. This point is considered significant to possible future product improvement proposals (PIP's) to the M188E1 propelling charge. The author regards this memorandum as an extension of the analysis begun by DRSAR-SA and reported in Ref c. The primary thrust of Ref c is to suggest appropriate statistical tests and sample sizes for determining the effect of charge temperature on the distribution of Δp derived from tests of the standard, unmodified M188E1 propelling charge. By contrast, this memorandum is principally concerned with the exposition of information contained in extant data.

6. Purpose of this Analysis

There are several objectives for analyzing the data in Ref a in greater detail. First, it is desired to distinguish the effect of charge preconditioning treatment on the interior ballistic results. Second, there is a need to identify all interior ballistic variables to which p_{max} is functionally related. Third, there is a need for a statistical (mathematical) model suitable for predicting catastrophic failure of this system. The author feels that these objectives have been met and, as a by-product of the analysis, computer programs exist which can facilitate future analyses of this sort. These are contained in Annex 2.

7. Methodology

The data analyzed consist of values of p_{max}, Δp, and ignition delay, Δt, obtained by firing unmodified M188E1/Z9 propelling charges which had been subjected to a particular preconditioning treatment and then temperature conditioned prior to firing. Typically, approximately twenty to thirty charges of a given treatment were fired sequentially before changing to charges with a different treatment. For purpose of analysis each

such homogenous batch was assigned a set number, for example, "secured cargo (cold)" is set C1. Shots on which one or more of the data items was lost were deleted from the sample.

8. The analysis performed on each of the sets and on various merged data sets consisted of the following:

a. calculating marginal statistics, i.e., mean and standard deviation of p_{max}, Δp, and Δt.

b. correlation analysis

(1) pairwise correlation coefficients of all of the variables

(2) point estimate of the slope of a simple (univariate) linear regression of p_{max} on Δp

(3) 95% confidence interval for (2)

(4) multiple linear regression treating p_{max} as a function of both Δp and Δt

(5) standard error of the estimate in (2) and conditional standard error in (4) above

c. stepwise polynomial regression of:

(1) p_{max} on Δp

(2) p_{max} on Δt

d. constrained quadratic regression of p_{max} on Δp with the constraint of positive slope for all Δp

e. analysis of residuals of p_{max} - Δp regression

(1) calculation of median rank statistics and estimation of the c.d.f. of the residuals

(2) test for normality of the residuals using the Finkelstein-Schafer test* and Lilliefors (K-S) test.

f. estimation and goodness-of-fit testing of the distribution of Δp

(1) estimation of a negative exponential distribution function

(2) estimation of the two parameters of a Weibull distribution by (a) matching moments and (b) maximum likelihood with estimation of the standard errors and 95% confidence intervals

(3) calculation of the Finkelstein-Schafer test statistics using the above hypothesized distributions

g. automatic plots of all regressions and all c.d.f.'s

h. calculation of the risk of a catastrophic failure. The theory for this calculation is derived in Annex 1.

9. Results and Conclusions

The data from both low and high temperature tests show that p_{max} exhibits a great deal of random variation from shot to shot which is not explained by its dependence on either Δp or Δt, separately or jointly. For example, the results of the secured cargo (cold) test, set C1, with sample of 27, show an average value of p_{max} of 37.596 ksi with a standard deviation of 0.772ksi. Linear regressions of p_{max} on Δp and p_{max} on Δt show slopes that are not significantly different from zero at a reasonable level of risk. In fact, the standard error of the estimate of p_{max} given Δp is 0.786 ksi. In this case, essentially no information concerning p_{max} is obtained from knowledge of Δp or Δt. If one were to perform a quadratic regression of p_{max} on Δp (in spite of the lack of a significant linear dependence), the residuals would have a standard deviation of 0.757 ksi.

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*The Finkelstein-Schafer test is a non-parametric goodness-of-fit test considerably more powerful than the modified Kolmogorov-Smirnov test. See Finkelstein, J.M. and Schafer, R.E., "Improved Goodness-of-fit Tests," Biometrika (1971), 58, 3, p. 641.

Thus, the fraction of the variance explained by the quadratic regression is less than 4% in this case. Similar types of statistics for the cold (-50 deg F) tests are shown in Table 1. Table 2 displays the analytical results for the hot (145 deg F) tests. The identity of the various statistics shown in Tables 1 and 2 will be presently clarified. The results of the correlation analysis of p_{max} on Δt and of Δp on Δt are not shown in Tables 1 or 2 since these correlations are not statistically different from zero. Additionally, partial correlation coefficients of p_{max} on Δp, given Δt, and of p_{max} on Δt, given Δp, are not very different from the corresponding unconditional correlations. Stated differently, knowledge of Δt does not enhance the predictability of either p_{max} or of Δp. Because of its lack of predictive capability, Δt will not be discussed further.

10. Inspection of the data shows that the joint occurence of exceptionally large values of Δp and p_{max} are derived from the charges preconditioned by sequential rough handling (seq. r.h.). This fact suggests that one pool the data selectively. Note that in both Tables 1 and 2 individual tests are analyzed separately and then progressively pooled or merged. For example in Table 2, the two secured cargo tests, sets H1 and H2, are merged and re-analyzed and then merged again with the hot cycle test, set H4, and analyzed again. Similarly, the sequential rough handling data, set H3 and set H5, is merged and analyzed as a single set.

11. Regression analyses of p_{max} on Δp at both cold and hot conditions show markedly different behavior between two different sets of data: (A) those representing charges which have been preconditioned by sequential rough handling, and (B) all others, i.e., those not subjected to sequential rough handling. The slope of a linear regression for set A (or any subset) is significantly positive (at the 97.5% confidence level) whereas that from set B is not. This observation applies at both cold and

hot conditions. This is seen from the results extracted from Tables 1 and 2 shown below in Table 3.

12. Additionally, stepwise polynomial regression performed on the Δp - p_{max} data consistently shows that set A requires* a quadratic or cubic regression to best fit the data, whereas set B does not support any regression above a linear one for the cold condition with a possible quadratic for the hot condition. The most striking aspect of these regressions is that if quadratic regressions are chosen for both sets A and B, the fraction of the variance explained by the regression function (R^2) distinguishes the two sets. This point is illustrated in Table 4. Note that a relatively tiny portion of the variance is explained by the regression for set B (non-seq. r.h.)--less than 3%--whereas almost half the variance is explained by set A (seq. r.h.).** Another indication of the difference in the $p_{max}(\Delta p)$ relationship for sets A and B (or selected subsets) is obtained by applying constrained quadratic regression to both sets. For set A the positive slope constraint is not binding and the mean squared residual for the constrained analysis is identical to that of the unconstrained regression. By contrast, for both cold and hot

---

*In this context the word "requires" implies a minimum mean squared residual for the required degree of the polynomial. Additionally, the value of R^2 does not change appreciably with further increases in the degree.

**This distinction in R^2 between sets A and B could be caused by more frequent occurences of large values of Δp in set A (rather than a difference in the $p_{max}(\Delta p)$ function). Then, assuming that p_{max} is a monotonically increasing function of Δp for both sets, a greater contribution to the variance of p_{max} would be made by the regression in set A. Were this hypothesis correct, one would also expect a greater variance of p_{max} from set A than from set B. In fact, this is not always the case. For the cold condition, the variance of set A is somewhat larger, with an F-statistic of 1.638, which is just significant at the 10% level; however, for the hot condition the F-statistic is 1.017 which is not plausibly significant at all. Therefore, this hypothesis is presently tenuous.

conditions for set B a constrained regression produces a greater mean squared residual than the corresponding unconstrained value, suggesting a different functional form for set B than for set A.

13. Aside from the differences revealed in sets A and B by regression analysis, a distinction also appears in the c.d.f. of Δp. Analyzing the data from each preconditioning treatment separately showed that, at a 5% risk level, all of the sets were accepted by the Finkelstein-Schafer (F-S) test as Weibull in form with the two parameters β and η in the c.d.f.:

$$F(x) = 1 - \exp[-(x/\eta)^{\beta}],*$$

estimated (where feasible) by maximum likelihood. After pooling all seq. r.h. data at a given charge temperature to form set A, and all other data to form set B, re-estimation of β and η -- $\hat{\beta}$ and $\hat{\eta}$ with the larger samples also showed that the distribution of sets A and B were Weibull but have different parameter values. The parameter estimates and their standard errors are shown in Table 5. Assuming asymptotic normality and applying a two-sided t-test to the difference:

$$\hat{\beta}\ (\text{set B}) - \hat{\beta}\ (\text{set A})\ ,$$

demonstrates that the Weibull shape parameter for set A (cold) is statistically distinguishable from that for set B (cold) with 99% confidence. Although the difference for the hot charge:

$$\hat{\beta}\ (\text{set B}) - \hat{\beta}\ (\text{set A})$$

is also positive, a t-test does not reveal a distinguishable difference.

14. However, for the cold charge condition, additional evidence of a difference in sets A and B with respect to the c.d.f. of Δp was obtained

*An alternative form of the distribution was also used:

$$F(x) = 1 - \exp - \lambda x^{\alpha}$$

with $\lambda = (1/\eta)^{\beta}$ and $\alpha = \beta$. Maximum likelihood estimates for α and λ and standard errors of $\hat{\alpha}$ and $\hat{\lambda}$ are found in Lloyd, D.K. and Lipow, R. Reliability (1962), pp. 177-181.

by merging sets A and B, re-estimating parameters and applying the F-S test. For the pooled data, the test rejected the Weigull hypothesis at a 95% confidence level. All of the above evidence strongly suggests that sequential rough handling occasionally produces a change in the structure of the M188E1 propelling charge which affects both the probability distribution of Δp and the functional relation $p_{max}(\Delta p)$.

15. Analysis of the residuals from a quadratic regression of p_{max} on Δp by F-S and K-S tests (95% confidence) consistently showed a normal (gaussian) c.d.f. The standard deviation of residuals, σ_z, and the regression coefficients are shown in Tables 1 and 2. Using the theory of Annex 1, these parameters were used to estimate the probability of a catastrophic failure in the M110A1 - M188E1 system. This risk is equated to the probability that p_{max} exceeds a critical value. The risks are shown in Table 6, in units of 10^{-6}, for sets A and B at both low and high temperature. Numerical uncertainty in the calculation of risk is shown under "integration relative error." This error is principally due to inaccuracy in the standard normal integral. Note that the critical value of p_{max} was assigned two values--50 and 53 ksi. Doubtless there is some uncertainty in p_{max} (critical) itself. This range in the critical value may be somewhat excessive but demonstrates the sensitivity of the risk to this parameter. Even for the larger value--53 ksi, which slightly exceeds an estimate by the BRL (Ref d)--the risk of failure is prohibitively large for charges which have been sequentially rough handled. For cold charges, where the risk is greatest, the probability of failure is 64 times the risk goal of 10^{-6}, using a critical pressure of 53 ksi.* However, without sequential rough handling the risk is substantially less than 10^{-6}.

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*As a representative example of the wartime consequence of a failure rate of 64 10^{-6} per shot, the following calculation is given. Suppose that an average of 200 weapons are operating in Europe for 90 days of combat. Let us suppose that the average consumption of M188 charges for this period is only 15 per weapon per day. Using these figures, about 17 catastrophic failures are expected in 90 days of combat.

TABLE 1. PARAMETER VALUES FOR RISK ASSESSMENT IN THE M110A1 S.P. HOW. USING THE M188E1 PROP. CHARGE AT -50 DEG. F.

Case Description	Sample Size	Regression Params. (ksi/ksi^n)			σ_z (ksi)	$\hat{\beta}$	$\hat{\eta}$ (ksi)	Resid. Mean Sqrs. (ksi) and R^2 for Degree:						95% Conf. Interval on Slope of Linear Reg. (ksi/ksi)		Slope (ksi/ksi)
								1		2		3				
		a_0	a_1	a_2				MS	R^2	MS	R^2	MS	R^2			
seq. rough hand. (cold) 28 Jan, set C3	15	36.944	0.15531	0.13410	0.6307	0.8940	0.6846	0.4403	0.3190	0.4641	0.3373	0.4544	0.4053	0.07	1.04	0.55
seq. rough hand., sets C2 and C3	27	36.798	0.34983	0.030178	0.6227	0.6211	0.4688			0.4201	0.4525	0.4205	0.4748	0.27	0.72	0.5
secured cargo (cold), set C1	27	37.344	1.0030	-0.52954	0.7568	1.0461	0.5226	0.6183	0.0021	0.6205	0.0385	0.6940	0.0548	-0.58	0.72	0.07
cold soak (-50^{0}F), set C4	23	37.255	0.33472	-0.42884	0.5138	1.7586	0.5248			0.2904	0.0066	0.2680	0.1289	-0.95	0.76	-0.09
no precond. (cold), set C5	10	37.768	-0.69356	-0.054821	0.4707	1.4711	0.6404	0.2494	0.3083	0.2848	0.3087	0.2805	0.4164	-1.74	0.17	-0.78
merge of sets C1, C4, C5	60	37.420	0.14045	-0.13593	0.6560	1.2750	0.5464	0.4389	0.0023	0.4455	0.0049	0.4529	0.0061	-0.51	0.35	-0.08
merged data from all cold tests, C1 - C5	87	37.238	0.061270	0.066785	0.6856	0.7930*	0.4902*	0.4844	0.1090	0.4813	0.1251	0.4869	0.1254	0.13	0.55	0.34

* The parameters from the merged data produce a distribution rejected by the Finklestein-Schafer test as not fitting the data.

TABLE 2. PARAMETER VALUES FOR RISK ASSESSMENT IN THE M110A1 S.P. HOW. USING THE M188E1 PROP. CHARGE AT 145 DEG. F.

Case Description	Sample Size	Regression Params. (ksi/ksi^n)			σ_z (ksi)	$\hat{\beta}$	$\sigma_{\hat{\beta}}$	$\hat{\eta}$ (ksi)	Resid. Mean Sqrs. (ksi) and R^2 for Degree: 1		2		3		95% Conf. Interval on Slope of Linear Reg. (km/km)		Slope of Linear Reg. (km/km)
		a_0	a_1	a_2					MS	R^2	MS	R^2	MS	R^2			
secured cargo (hot), set H1	26	42.393	-0.048854	0.006554	0.6701	1.278	0.1940	1.511			0.4881	0.0016			-0.28	0.23	-0.02
secured cargo (hot), set H2	21	42.586	-0.15656	0.034356	0.7401	1.128	0.1851	2.452			0.6086	0.0458			-0.17	0.31	0.07
seq. rough hand. (hot), set H3	12	42.528	-2.5825	1.6719	0.5112	1.783	0.4311	0.8204			0.3194	0.1871			-1.03	0.83	-0.10
hot cycle, set H4	29	42.070	-0.53518	0.17826	0.7013	1.310	0.1914	1.267	0.5455	0.0074	0.5297	0.0718			-0.25	0.39	0.07
seq. rough hand. (hot), set H5	17	42.005	0.18880	0.04820	0.5562	1.011	0.1923	1.348			0.3535	0.5327			0.20	0.67	0.43
secured cargo (hot), sets H1 + H2	47	42.507	-0.14951	0.033471	0.6981	1.200	0.1323	1.811	0.5091	0.0040	0.5095	0.0253			-0.13	0.20	0.03
seq. rough hand. (hot), sets H3 + H5	29	41.888	0.11657	0.067996	0.5839	1.081	0.1521	1.331			0.3672	0.4290	0.3817	0.4292	0.23	0.66	0.45
merge of sets H1, H2, H4 non-seq. r. h.	76	42.223	-0.11881	0.037625	0.7551	1.222	0.1065	1.570	0.5883	0.0111	0.5857	0.0287			-0.08	0.22	0.07
merged data from all hot tests, H1 - H5	105	42.118	-0.038302	0.040708	0.7341	1.176	0.0872	1.471			0.5495	0.0774	0.5520	0.0823	0.04	0.29	0.16

TABLE 3. COMPARISON OF REGRESSION ESTIMATES FROM TWO SETS OF M188E1 PROPELLING CHARGES HAVING DIFFERENT TREATMENTS

Set/ Treatment	Charge Temperature (deg F)			
	-50		145	
	slope (ksi/ksi)	95% C.I.	slope (ksi/ksi)	95% C.I.
Set A, seq. r.h.	0.50	0.27, 0.72	0.45	0.23, 0.66
Set B, non-seq. r.h.	-0.08	-0.51, 0.35	0.07	-0.08, 0.22

TABLE 4. FRACTION OF VARIANCE EXPLAINED BY A QUADRATIC REGRESSION OF PMAX ON DELP (R^2), DISTINGUISHED BY CHARGE TREATMENT

Set/ Treatment	Charge Temperature (deg F)	
	-50	145
Set A seq. r.h.	0.4525	0.4290
Set B non-seq. r.h.	0.0049	0.0287

TABLE 5. WEIBULL SHAPE PARAMETER FOR THE DISTRIBUTION FUNCTION OF ΔP STATISTICS, DISTINGUISHED BY CHARGE TREATMENT

Set/ Treatment	Charge Temperature (deg F)			
	-50		145	
	$\hat{\beta}$	$\sigma_{\hat{\beta}}$	$\hat{\beta}$ *	$\sigma_{\hat{\beta}}$ *
Set A, seq. r.h.	0.6211	0.132	1.081	0.152
Set B, non-seq. r.h.	1.275	0.154	1.222	0.106

*maximum likelihood estimates used

TABLE 6. RISK OF CATASTROPHIC FAILURE IN THE M110A1-M188E1/Z9 SYSTEM

Set/ Treatment	Charge Temp (°F)	Crit p max (ksi)	Risk (10^{-6})	Integration Rel. Error
Set A,	-50	50	134	0.026
seq. r. h.		53	64	0.016
	145		22	$<1.3\ 10^{-3}$
Set B,	-50		0	$< 10^{-4}$
w/o seq. r. h.	145		0	$< 10^{-3}$

DRSAR-SAM
SUBJECT: Analysis of Interior Ballistic Safety Test Data from the M110A1 Using the M188E1 Propelling Charge Subjected to Several Treatments

16. Recommendations

In view of the statistical differences in interior ballistics observed between M188E1 charges which have been sequentially rough handled (set A) and those which have not (set B), it is recommended that the PM110E2 pursue the present testing program with a goal of identifying in detail the physical causes for the difference. The author recently was informed that defects such as torn propellant bags and cracked igniter tubes had been detected in rough handled charges. This physical evidence supports the conclusions based upon our analysis that physical (geometrical) changes occur due to sequential rough handling. Clearly, a product improvement must be initiated if the risk of a catastrophic failure is to be reduced to 10^{-6}. Until this improvement is accomplished and demonstrated, it is prudent to restrict the firing of M188E1 charges to those which demonstrably have not been rough handled. It is further recommended that a firing test program be conducted using unmodified, product-improved charges which have been sequentially rough handled to assess risk of failure. The present evidence suggests subtile effects on the c.d.f. of Δp and on the $p_{max}(\Delta p)$ relationship due to igniter/charge geometry. Therefore, inferences with respect to firing safety of a particular charge drawn from grossly altered or different charges are shakey at best.

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ANNEX 1.

Probability that p_{max} Exceeds a Critical Value

The risk of having a catastrophic failure of some sort in the system, e.g., inbore premature or breech blow, is equated to the probability that the maximum chamber pressure exceeds a critical value. Having investigated several alternative mathematical models for the random variable p_{max} and having selected an optimum, one assumes that this model can be extrapolated to large values of Δp, i.e. that the statistics at extreme pressure have the same mathematical form as that determined for lower values. The necessary assumptions for the calculation of this risk are given below.

Assumptions

(1) In the standard M188E1 propelling charge subjected to the expected rough handling, the maximum chamber pressure depends only on the absolute negative pressure differential during ignition, i.e.,

$$p_{max} = p_{max}(\Delta p),$$

(2) The dependence in (1) is quadratic at every charge temperature.

(3) For any value of Δp, p_{max} has an additive, random component which is normally (gaussian) distributed with mean zero and constant variance and is uncorrelated with Δp.

(4) The pressure differential, Δp, is stochastic having a two-parameter Weibull distribution.

(5) The critical value of p_{max} is a constant, viz. 53 ksi.

Derivations

Notationally, let y be the dependent variable, x the independent variable--Δp, in this case--and Z the gaussian noise. Then, the random variable Y is given by

$$Y = a_0 + a_1 X + a_2 X^2 + Z \ , \tag{1}$$

with X having the p.d.f. $f_X(x)$ and with Z having the p.d.f. $f_Z(z)$. Notationally,

$$F_X(x) = \int_0^x f_X(u)du \ .$$

Using assumption (1), (2), and (3), the probability that Y is less than a particular value y is

$$P\{Y < y\} = F_y(y)$$

$$= P\{a_0 + a_1X + a_2X^2 + Z < y\} \tag{2a}$$

$$F_y(y) = \int_{\text{all } x} P\{x < X < x + dx\} \cdot P\{Z < y - a_0 - a_1x - a_2x^2\} \; . \tag{2b}$$

$$F_y(y) = \int_0^\infty f_x(x)\; F_z(y - a_0 - a_1x - a_2x^2)dx \; . \tag{3}$$

By assumption (4),

$$f_x(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-(x/\eta)^\beta} \tag{4}$$

Assumption (3) yields

$$F_z(z) = \frac{1}{\sqrt{2\pi}\;\sigma_z} \int_{-\infty}^{z} e^{-t^2/(2\sigma_z)}\; dt \; . \tag{5}$$

The risk of exceeding the critical value y_c is given from (3, 4, 5) as $1 - F_y(y_c)$. However, the above equations do not have an analytic solution. Therefore, numerical integration is employed using parameter values given in Table 1.

The integrand in equation (3):

$$g(x) = f_x(x)\; F_z(y - a_0 - a_1x - a_2x^2) \tag{6}$$

can be evaluated using existing subroutines. Integration of (3) is accomplished using the trapezoidal rule:

$$F_y \approx \frac{g(x_1)h}{2} + \frac{h}{2}\,\Sigma_{i=1}^{n}\; g(x_i) + g(x_{i+1}) \; , \tag{7}$$

with step size h given by

$$h = \min(0.005\ \sigma_z,\ 0.005\ \eta) \tag{8}$$

with

$$x_1 = 0$$

and with truncation at x_n where

$$(x_n/\eta)^\beta > -\ln \varepsilon$$

and where ε is the largest permissible error in the integrand. Generally, a suitable value of $\varepsilon \sim 10^{-7}$.

Since the desired computational quantity is actually the risk ρ

$$\rho = 1 - F_y(y_c)\ , \tag{9}$$

rather than first calculate $F_y(y_c)$ it is numerically more accurate to solve the following equation directly. From (3),

$$\rho = \int_0^\infty f_x(x)[1 - F_z(z)]dx \tag{10a}$$

with

$$z = y_c - \max(0,\ a_0 + a_1x + a_2x^2)\ . \tag{10b}$$

The numerical approximation is

$$\rho \simeq \frac{g'(x_1)h}{2} + \frac{h}{2}\ \Sigma_{i=1}^n g'(x_i) + g'(x_{i+1})$$

where

$$g'(x) = f_x(x)[1 - F_z(z)]\ . \tag{11}$$

Simultaneous integration of (3) via (7) provides an estimate of the integration error.

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ANNEX 2.

COMPUTER PROGRAMS USED IN A STATISTICAL ANALYSIS OF INTERIOR BALLISTIC SAFETY TEST DATA

Several source program listings are given in this annex. All programs are written in FORTRAN 4. The first main program and associated subroutines perform all of the analyses outlined in the body of this memorandum exclusive of constrained regression and risk analysis, which are listed separately. Comments provided in the listings indicate the program operations to be performed. The statistical analysis main program and subroutines require approximately 250 K bytes of core storage using the WATFIV diagnostic compiler with the IBM 360 system. A sample run consisting of the analysis of several data sets follows the program listing. Input control parameters are echoed in the output.

The second main program presented here performs a quadratic regression constrained to have positive slope for all values of the independent variable. Examples of program output follow the listing.

The third main program calculates the risk of a catastrophic failure in the M110A1-M188E1 system under the conditions implied by the input parameters. The algorithm of Annex 1 is employed in this calculation. Several examples are provided.

```
C     STATISTICAL ANALYSIS OF INTERIOR BALLISTIC DATA--PMAX,DELP,DELAY
C     UNITS:  PMAX (KSI), DELP (PSI), DELAY (MS)
C
C     THIS PROGRAM CALCULATES THE MARGINAL STATISTICS AND
C     CORRELATIONS OF THE MAX CHAMBER PRESSURE, PMAX, THE
C     ABSOLUTE PRESSURE DIFFERENTIAL ACROSS THE CHAMBER, DELP, AND
C     THE IGNITION DELAY, DELAY.  REGRESSION ANALYSES OF PMAX ON DELP
C     AND OF PMAX ON DELAY ARE CALCULATED TO A PRESCRIBED DEGREE.
C     AN ANALYSIS OF RESIDUALS FOR NORMALITY IS PERFORMED USING BOTH
C     THE LILLIEFORS (K-S) TEST AND THE FINKELSTEIN-SCHAFER TEST  .
C     THE DATA ARE ORDERED FROM SMALLEST TO LARGEST FOR EACH
C     VARIABLE AND SAMPLE C.D.F. S ARE PRINTED AND PLOTTED.  FINALLY,
C     THE FINKELSTEIN-SCHAFER TEST IS APPLIED TO A BEST ESTIMATE
C     OF THE PROBABILITY DISTRIBUTION FUNCTION OF DELP.
C
C     INPUTS:
C     CARD 1    DESCRIPTIVE ALPHAMERIC TITLE
C     CARD 2    SAMPLE SIZE, PLOT SWITCH, AND CRIT. VALUE AND RISK FOR F-S TEST
C     CARD 3FF ENTRIES IN THE DATA ARRAY:  PMAX(I), DELP(I), DELAY(I), I=1,NSAMP
C     OUTPUTS:
C     1.        MEAN VALUES
C     2.        STD. DEVS. OF  PMAX, DELP, DELAY
C     3.        CORRELATION COEFS., LINEAR SLOPES, AND STD. ERRORS
C     4.        REGRESSION COEFFICIENTS ETC. FOR PMAX ON DELP AND PMAX ON DELAY
C     5.        ORDER STATISTICS FOR THE DISTRIBUTION FUNCTIONS OF DATA VARS.
C     6.        GOODNESS-OF-FIT TEST OF A PROPOSED DISTRIBUTION--DISTF--
C               USING THE FINKLESTEIN-SCHAFER TEST STATISTIC.
C     SEE PAGES 336 FF, MANN, N. ET AL., METHODS FOR STATISTICAL
C     ANALYSIS OF RELIABILITY AND LIFE DATA, C. 1974.
C
      DIMENSION TITLE(20),DATA(400,3),REGM(400,4),PMAX(400),DELP(400),
     1 DELAY(400),COVM(3,3),RHO(3,3),SUM(9),CI(4),CJ(4),IVCHAR(I0)
      EQUIVALENCE (DATA(1,1),PMAX(1)),(DATA(I,2),DELP(1)),
     1 (DATA(1,3),DELAY(1))
      DATA ICHAR,ISTAR/'0','*'/
      DATA IVCHAP/'*','2','3','4','5','6','7','8','9','X'/
    1 CONTINUE
      READ (5,10,END=3) TITLE
   10 FORMAT(20A4)
      WRITE (6,12) TITLE
   12 FORMAT(1H1,20A4)
      READ (5,14) NSAMP,IPLOT,NDEG,FSTST,FSRISK
   14 FORMAT(3I3,1X,2F10.0)
      NDP1=NDEG+1
      IF(FSTST.EQ.0.0) FSTST=(FLOAT(NSAMP)/3.03)**0.37
      IF(FSTST.LT.0.0) FSTST=(FLOAT(NSAMP)/2.506)**0.383
C
C     DEFAULT CRITICAL VALUES FOR THE F-S TEST ARE OBTAINED BY PROVIDING
C     ZERO FOR A 10% RISK AND ANY NEGATIVE VALUE FOR A 5% RISK.
C     THE ABOVE DEFAULT APPROXIMATIONS ARE GOOD FOR A SAMPLE GT. 6.
C
      WRITE (6,16) NSAMP,FSTST,FSRISK,NDEG
   16 FORMAT(1H0,'SAMPLE = ',I3,'  F-S CRITICAL VALUE = ',F10.3,4X,
     1 ' F-S LEVEL OF RISK = ',F10.4,4X,' DEG. OF POLY. REGRES. =',I3)
C
C     DATA INPUT SECTION
C     READ DATA ARRAYS
```

```
C
      DO 15 I=1,NSAMP
      READ (5,17) PMAX(I),DELP(1),DELAY(1)
   17 FORMAT(3F10.0)
      IF(PMAX(I).GT.57.0.OR.DELAY(I).GT.1000..OR.DELP(I).GT.9000.) GO TO
     1 4
   15 CONTINUE
C     END OF DATA INPUT SECTION
C
C     CALCULATE MARGINAL STATISTICS
C
      FN=NSAMP
      DO 18 J=1,3
      SUM(J)=0.0
      DO 20 I=1,3
      COVM(1,J)=0.0
      RHO(I,J)=0.0
   20 CONTINUE
   18 CONTINUE
      DO 22 J=1,3
      DO 24 1=1,NSAMP
      SUM(J)=SUM(J)+DATA(I,J)
   24 CONTINUE
      SUM(J)=SUM(J)/FN
   22 CONTINUE
      DO 26 J=1,3
      DO 28 1=1,NSAMP
      REGM(I,J)=DATA(1,J)-SUM(J)
   28 CONTINUE
   26 CONTINUE
C
C     WRITE MEAN VALUES
C
      WRITE (6,30) SUM(1),SUM(2),SUM(3)
   30 FORMAT(1H0,3X,12HAVG PMAX,KSI,3X,12HAVG DELP,PS1,3X,
     1 12HAVG TMDEL,MS/1H ,F15.4,2F15.1)
      DO 32 1=1,3
      DO 34 J=1,3
      DO 36 K=1,NSAMP
      COVM(I,J)=COVM(I,J)+REGM(K,I)*REGM(K,J)
   36 CONTINUE
      COVM(I,J)=COVM(I,J)/FLOAT(NSAMP-1)
   34 CONTINUE
   32 CONTINUE
      DO 38 I=1,3
      DO 40 J=1,3
      RHO(I,J)=COVM(I,J)/SQRT(COVM(I,1)*COVM(J,J))
   40 CONTINUE
   38 CONTINUE
      SUM(4)= SQRT(COVM(1,1))
      SUM(5)= SQRT(COVM(2,2))
      SUM(6)= SQRT(COVM(3,3))
      FNP1=FLOAT(NSAMP+1)
      FNM1=FLOAT(NSAMP-1)
C
C     CALCULATE THE SLOPE OF A SIMPLE LINEAR REGRESSION OF PMAX ON DELP
C
      SLOPE=SUM(4)*RHO(1,2)/SUM(5)
```

```
C
C     CALCULATE THE STANDARD ERROR OF THE ESTIMATE OF PMAX, GIVEN DELP
C
      SEPMAX=SUM(4)*SQRT((1.-RHO(1,2)**2)*FNM1/(FNM1-1.))
C
C     STD. DEV. OF THE SLOPE AND ASSOCIATED 95% CONFIDENCE INTERVAL
C
      SDSLOP=SEPMAX/SUM(5)/SQRT(FNM1)
C
C     CALCULATE THE T-STATISTIC AND 95% CONFIDENCE LIMITS
C
      NU=NSAMP-2
      T=STUDNT(0.025,NU)
      SLO=SLOPE-T*SDSLOP
      SHI=SLOPE+T*SDSLOP
C
C     CALCULATE PARTIAL CORRELATION COEFFICIENTS
C
      RY1G2=(RHO(2,1)-RHO(3,1)*RHO(3,2))/ SQRT((1.-RHO(3,1)**2)*
     1 (1.-RHO(3,2)**2))
      RY2G1=(RHO(3,1)-RHO(2,1)*RHO(3,2))/ SQRT((1.-RHO(2,1)**2)*
     1 (1.-RHO(3,2)**2))
C
C     RY1G2 IS THE PARTIAL CORRELATION OF PMAX ON DELP, GIVEN DELAY
C     RY2G1 IS THE PARTIAL CORRELATION OF PMAX ON DELAY, GIVEN DELP
C
      VDTDP=RY1G2**2
      VDTDLY=RY2G1**2
      WRITE (6,42) SUM(4),SUM(5),SUM(6)
   42 FORMAT(1H0,2X,13HSD:  PMAX,KSI,7X,8HDELP,PSI,7X,8HDELAY,MS/1H ,
     1 3F15.4/1H0,'CORRELATION MATRIX:'/1H0,11X,4HPMAX,11X,4HDELP,10X,
     2 5HDELAY)
C
C     WRITE CORRELATION MATRIX
C
      WRITE (6,44) ((RHO(I,J),J=1,3),I=1,3)
   44 FORMAT(1H ,3F15.4)
      WRITE (6,43) SLOPE,SEPMAX,SDSLOP,SLO,SHI,T
   43 FORMAT(1H0,'ANALYSIS BASED ON SIMPLE LINEAR REGRESSION OF PMAX ON
     1DELP:'/1H0,'SLOPE =',F10.5,' KSI/PSI',5X,'STD. ERR. OF EST. OF PMA
     2X GIVEN DELP =',F10.5,' KSI'/1H0,'STD. DEV. OF SLOPE =',F10.5,5X,
     3'95% CONF. LIMITS:  ',2F10.5,5X,'T-STATISTIC (CRITICAL) =',F10.5)
      WRITE (6,45) RY1G2,RY2G1,VDTDP,VDTDLY
   45 FORMAT(1H0,'PARTIAL CORRELATION COEFS. WITH DEPENDENCE OF PMAX ON
     1DELP AND DELAY ASSUMED'/1H0,15H DELP, GIVEN DT,15H DT, GIVEN DELP/
     2 1H ,2F15.4/1H0,'FRACTION OF VARIANCE DUE TO DELP =',F15.4/
     3 1H0,'FRACTION OF VARIANCE DUE TO DELAY =',F15.4)
C
C     FILL REGRESSION MATRIX FOR REGRESSION ANALYSIS OF PMAX ON DELP
C
      DO 48 J=1,NDEG
      DO 50 I=1,NSAMP
      REGM(I,J)=(DELP(I))**J
   50 CONTINUE
   48 CONTINUE
      DO 52 I=1,NSAMP
      REGM(I,NDP1)=PMAX(I)
   52 CONTINUE
```

```
      CALL MLR(REGM,CI,400,4,NSAMP,NDP1,I,.TRUE  RSQ)
C
C     CALCULATE FRACTION OF VARIANCE EXPLAINED BY REGRESSION
      FRACT=RSQ
C
C     WRITE REGRESSION COEFFICIENTS
C
      WRITE (6,54) (CI(J),J=1,NDP1)
   54 FORMAT(1H0,'REGRESSION COEFS. FOR PMAX ON DELP:'/1H0,9X,
     1 6HZEROTH,10X,5HFIRST,12X,3H2ND,12X,3H3RD/1H ,4E15.5)
      WRITE (6,56) FRACT
   56 FORMAT(1H0,'FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION:',
     1 F15.5)
C
C     CALCULATE, SORT, AND PRINT  (PLOT) THE RESIDUALS
C
      RMAX=-1.E20
      RMIN=1.E20
      SR=0.
      SSR=0.
      DO 57 I=1,NSAMP
      REGM(I,1)=PMAX(I)-CI(1)
      DO 59 J=2,NDP1
      REGM(I,1)=REGM(I,1)-CI(J)*DELP(I)**(J-1)
   59 CONTINUE
      SR=SR+REGM(I,1)
      SSR=SSR+(REGM(I,1))**2
      IF(REGM(I,1).GT.RMAX) RMAX=REGM(I,1)
      IF(REGM(I,1).LT.RMIN) RMIN=REGM(I,1)
   57 CONTINUE
      SR=SR/FN
      SSR=SQRT((SSR/FN-SR**2)*FN/FNM1)
      IF(IPLOT.EQ.1) CALL PSCALE(1,1,RMIN,RMAX,XMIN,XMAX)
      IF(IPLOT.EQ.1) CALL GPPLOT(1,XMIN,XMAX,0.,1.)
      NM1=NSAMP-1
      DO 159 I=1,NM1
      IP1=I+1
      DO 160 II=IP1,NSAMP
      IF(REGM(I,1).LE.REGM(II,1)) GO TO 161
      HOLD=REGM(I,1)
      REGM(I,1)=REGM(II,1)
      REGM(II,1)=HOLD
  161 CONTINUE
  160 CONTINUE
  159 CONTINUE
C
C     END OF SORT OF RESIDUALS
C
      WRITE (6,162)
  162 FORMAT(1H1,'DISTRIBUTION OF RESIDUALS FROM PMAX-DELP REGRESSION'/
     1 1H0,5HINDEX,10H RESID,KSI,10H  MED RANK,10H NORM D.F.,
     2 5X,5HDSTAR,5X,5HSSTAR)
C
C     INITIALIZE LILLIEFORS TEST STATISTIC AND CALCULATE THE CRITICAL VALUE
C
      SRFNP1=SQRT(FLOAT(NSAMP+1))
      IF(FSRISK.LE.0.01) CLFORS=1.04/SRFNP1
      IF(FSRISK.EQ.0.05) CLFORS=0.881/SRFNP1
```

```
      IF(FSRISK.GE.0.1) CLFORS=0.805/SRFNP1
      TLFORS=0.0
      SSTAR=0.0
      DO 163 I=1,NSAMP
      Y=FLOAT(I)/FNP1
      X=REGM(I,1)
      Z=SNORM((X-SR)/SSR)
      F1=FLOAT(I)/FN
      F2=FLOAT(I-1)/FN
      T1=F1-Z
      T2=Z-F2
      DSTAR=AMAX1(T1,T2)
      IF(DSTAR.GT.TLFORS) TLFORS=DSTAR
      SSTAR=SSTAR+ABS(DSTAR)
      IF(IPLOT.EQ.1) CALL GPLOD1(X,Y,ISTAR)
      WRITE (6,164) I,X,Y,Z,DSTAR,SSTAR
  164 FORMAT(1H ,I5,5F10.4)
  163 CONTINUE
      IF(SSTAR.GT.FSTST) GO TO 175
      WRITE (6,180) SSTAR,FSTST
  180 FORMAT(1H0,'RESIDUALS ARE PLAUSIBLY GAUSSIAN.'/1H0,
     1 'SSTAR IN F-S TEST,',F10.4,' IS LESS THAN THE CRITICAL VALUE,',
     2 F10.4)
      GO TO 172
  175 CONTINUE
      WRITE (6,182) SSTAR,FSTST
  182 FORMAT(1H0,'RESIDUALS ARE NOT PLAUSIBLY GAUSSIAN.'/1H0,
     1 'SSTAR IN F-S TEST,',F10.4,' EXCEEDS THE CRITICAL VALUE,',F10.4)
  172 CONTINUE
      IF(TLFORS.GT.CLFORS) GO TO 184
      WRITE (6,190) TLFORS,CLFORS,FSRISK
  190 FORMAT(1H0,'LILLIEFORS (K-S) TEST SHOWS RESIDUALS ARE GAUSSIAN.'/
     1 1H ,'TEST STATISTIC ',F10.4,' IS LESS THAN THE CRITICAL VALUE ',
     2 F10.4,' WITH A RISK OF ',F10.4)
      GO TO 173
  184 CONTINUE
      WRITE (6,192) TLFORS,CLFORS,FSRISK
  192 FORMAT(1H0,'LILLIEFORS (K-S) TEST SHOWS RESIDUALS ARE NOT GAUSSIAN
     1.'/1H ,'TEST STATISTIC ',F10.4,' IS GREATER THAN THE CRITICAL VALU
     2E ',F10.4,' WITH A RISK OF ',F10.4)
  173 CONTINUE
      WRITE (6,165) SR,SSR
  165 FORMAT(1H0,'MEAN AND STD. DEV. OF RESIDUALS (KSI):',2F15.5)
      IF(IPLOT.NE.1) GO TO 170
      WRITE (6,166)
  166 FORMAT(1H1,'PLOT OF CUMULATIVE DISTRIBUTION FUNCTION OF PMAX RESID
     1UALS')
      CALL GPPRNT
  170 CONTINUE
C
C     FILL REGRESSION MATRIX FOR REGRESSION OF PMAX ON DELAY
C
      DO 58 J=1,NDEG
      DO 60 I=1,NSAMP
      REGM(I,J)=(DELAY(I))**J
   60 CONTINUE
   58 CONTINUE
      DO 62 I=1,NSAMP
```

```
      REGM(I,NDPI)=PMAX(I)
   62 CONTINUE
      CALL MLR(REGM,CJ,400,4,NSAMP,NOP1,1,.TRUE.,S,RSQ)
      FRACT=RSQ
C
C     WRITE REGRFSSION COEFFICIENTS
C
      WRITE (6,64) (CJ(J),J=I,NDPI)
   64 FORMAT(1H0,'REGRESSION COEFS. FOR PMAX ON DELAY:'/IH0,9X,
     I 6HZEROTH,10X,5HFIRST,I2X,3H2ND,I2X,3H3RD/1H ,4EI5.5)
      WRITE (6,56) FRACT
C
C     PRESERVE UNSORTED DATA
C
      DO 19 I=I,NSAMP
      REGM(I,1)=PMAX(I)
      REGM(I,2)=DELP(I)
      REGM(I,3)=DELAY(I)
   19 CONTINUE
C
C     REGRESSION ANALYSIS COMPLETE; NOW SORT DATA
C
      NMI=NSAMP-I
      00 66 J=I,3
      DO 68 I=I,NMI
      IPI=I+I
      00 70 II=IPI,NSAMP
      IF(DATA(I,J).LE.DATA(II,J)) GO TO 72
      HOLO=DATA(I,J)
      OATA(I,J)=DATA(II,J)
      OATA(II,J)=HOLD
   72 CONTINUE
   70 CONTINUE
   68 CONTINUE
C     END OF I LOOP
C
   66 CONTINUE
C
C     END OF VARIABLES (J)-LOOP AND END OF SORT
C
C     WRITE SORTED STATISTICS
C
      WRITE (6,74)
   74 FORMAT(1H0,5HINDEX,IOH  MED RANK.7X,8HPMAX,KSI,7X,8HDELP,PSI,
     I 7X,8HOELAY,MS)
      00 78 I=1,NSAMP
      RANK=FLOAT(I)/FLOAT(NSAMP+I)
      WRITE (6,76) I,RANK,PMAX(I),DELP(I),DELAY(I)
   76 FORMAT(IH ,2X,I3,F10.4,FI5.4,2FI5.2)
   78 CONTINUE
      SSTAR=0.0
      FNFI=NSAMP+I
      IF(IPLOT.NE.I) GO TO 94
C
C     PLOT REGRESSION FUNCTION AND DATA OF PMAX ON DELP
C
C
C     DETERMINE PLOT LIMITS
```

```
C
      CALL PSCALF(1,1,0.,DELP(NSAMP),XMIN,XMAX)
      CALL PSCALF(3,2,PMAX(I),PMAX(NSAMP),YMIN,YMAX)
      CALL GPPLOT(2,XMIN,XMAX,YMIN,YMAX)
      X=0.0
      DX=DELP(NSAMP)/100.
      DO 71 I=1,100
      X=X+DX
      Y=0.0
      DO I71 J=1,NDP1
      Y=Y+CI(J)*X**(J-I)
  I71 CONTINUE
      CALL GPLOD1(X,Y,ICHAR)
   7I CONTINUE
C
C     LOAD SECOND LAYER
C
      DO 73 I=I,NSAMP
      X=REGM(I,2)
      Y=REGM(I,1)
      CALL GPLOD2(X,Y)
   73 CONTINUE
C
C     PLOT OUT REGRESSION
C
      WRITE (6,75)
   75 FORMAT(1H1,'REGRESSION OF PMAX ON DELP')
      CALL GPCONV(IVCHAR,10)
      CALL GPPRNT
C
C     PLOT REGRESSION FUNCTION AND DATA OF PMAX ON DELAY
C
      CALL PSCALE(3,1,DELAY(1),DELAY(NSAMP),XMIN,XMAX)
      PMAXLO=PMAX(1)-0.1*(PMAX(NSAMP)-PMAX(I))
      CALL PSCALE(3,2,PMAXLO,PMAX(NSAMP),YMIN,YMAX)
      CALL GPPLOT(2,XMIN,XMAX,YMIN,YMAX)
      X=DELAY(1)
      DX=(DELAY(NSAMP)-DELAY(1))/100.
      DO 77 I=1,100
      X=X+DX
      Y=0.0
      DO 177 J=1,NDP1
      Y=Y+CJ(J)*X**(J-1)
  177 CONTINUE
      CALL GPLOD1(X,Y,ICHAR)
   77 CONTINUE
C
C     LOAD SECOND LAYER
C
      DO 79 I=1,NSAMP
      X=REGM(I,3)
      Y=REGM(I,1)
      CALL GPLOD2(X,Y)
   79 CONTINUE
C
C     PLOT OUT REGRESSION
C
      WRITE (6,81)
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   81 FORMAT(1H1,'REGRESSION OF PMAX (KSI) ON DELAY (MS)')
      CALL GPCONV(IVCHAR,10)
      CALL GPPRNT
      CALL PSCALE(1,1,0.0,DELP(NSAMP),XMIN,XMAX)
C
C     PROVIDE CONTROL INFORMATION FOR PLOTTING
C
      CALL GPPLOT(1,0.,XMAX,0.,1.)
C
C     ESTIMATE THE PARAMETERS OF THE C.D.F. OF DELP
C
      CALL DIST1(SUM(2),SUM(5),NSAMP,DELP)
C
C     LOAD THE PLOT CHARACTERS FOR THE CDF INTO A PLOTTING ARRAY
C
      X=0.0
      DX=DELP(NSAMP)/100.
      DO 93 I=1,100
      X=X+DX
      CALL DISTF(X,Y)
      CALL GPLOD1(X,Y,ICHAR)
   93 CONTINUE
   94 CONTINUE
C
C     APPLY THE FINKLESTEIN-SCHAFER TEST AND LILLIEFORS (K-S) TEST
C
C     WRITE HEADINGS
C
      WRITE (6,90)
   90 FORMAT(1H0,5HINDEX,2X,8HDELP,KSI,7X,3HCDF,5X,5HDSTAR,5X,5HSSTAR)
      TLFORS=0.0
      DO 80 I=1,NSAMP
      F1= FLOAT(I)/FN
      F2= FLOAT(I-1)/FN
      X=DELP(I)
      Y=FLOAT(I)/FNP1
      IF(IPLOT.EQ.1) CALL GPLOD1(X,Y,ISTAR)
      CALL DISTF(X,CDFX)
      T1=F1-CDFX
      T2=CDFX-F2
      DSTAR=AMAX1(T1,T2)
      IF(DSTAR.GT.TLFORS) TLFORS=DSTAR
      SSTAR=SSTAR+ABS(DSTAR)
      WRITE (6,92) I,X,CDFX,DSTAR,SSTAR
   92 FORMAT(1H ,2X,I3,4F10.4)
   80 CONTINUE
      IF(SSTAR.GT.FSTST) GO TO 84
      WRITE (6,82) SSTAR
   82 FORMAT(1H0,'SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F. USING THE
     1F-S TEST',' S STAR = ',F10.4)
      GO TO 88
   84 CONTINUE
      WRITE (6,86) SSTAR
   86 FORMAT(1H0,'SAMPLE IS NOT PLAUSIBLE FROM THE HYPOTH. C.D.F. USING
     1THE F-S TEST'/1H0,'S STAR = ',F10.4)
   88 CONTINUE
      IF(TLFORS.GT.CLFORS) GO TO 284
      WRITE (6,290) TLFORS,CLFORS,FSRISK
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  290 FORMAT(1H0,'LILLIEFORS (K-S) TEST SHOWS SAMPLE IS PLAUSIBLE FROM T
     1HE HYPOTH. C.D.F.'/1H ,'TEST STATISTIC ',F10.4,' IS LESS THAN THE
     2CRITICAL VALUE ',F10.4,' WITH A RISK OF ',F10.4)
      GO TO 288
  284 CONTINUE
      WRITE (6,292) TLFORS,CLFORS,FSRISK
  292 FORMAT(1H0,'LILLIEFORS (K-S) TEST SHOWS THAT SAMPLE IS NOT PLAUSIB
     1LE FROM THE HYPOTH. C.D.F.'/1H ,'TEST STATISTIC ',F10.4,' IS GREAT
     2ER THAN THE CRITICAL VALUE ',F10.4,' WITH A RISK OF ',F10.4)
  288 CONTINUE
      IF(IPLOT.NE.1) GO TO 1
C
C     PLOT DISTRIBUTIONAL DATA
C
      WRITE (6,89)
   89 FORMAT(1H1,'PLOT OF HYPOTH. AND SAMPLE C.D.F.S OF DELP')
      CALL GPPRNT
      GO TO 1
    4 CONTINUE
C
C     WRITE ERROR MESSAGE AND STOP
C
      WRITE (6,400) PMAX(I),DELP(I),DELAY(I)
  400 FORMAT(1H0,'ERROR IN INPUT--PMAX,DELP,DELAY:  ',3F15.4)
    3 CALL EXIT
      STOP
      END
C     ************************** DIST1 ******************************
      SUBROUTINE DIST1(XMEAN,XSTD,N,T)
C
C     ESTIMATION OF PARAMS OF THE TWO-PARAM WEIBULL C.D.F.
C
C     PARAMETERS ARE ESTIMATED BY THE METHOD OF MATCHING
C     MOMENTS FROM THE GIVEN MEAN AND STANDARD DEVIATION.
C     ALTERNATIVE ESTIMATES USING MAX LIKELIHOOD ARE ALSO OBTAINED
C
      DIMENSION T(400)
      A1=XMEAN
      A2=XMEAN**2+XSTD**2
      A=A2/A1**2
      BETA1=2.0
      DBETA=0.1
C
C     ITERATE FOR BETA (SHAPE PARAM)
C
      BETA=BETA1
      KOUNT=0
      NREV=0
    1 CONTINUE
C
C     RETURN OF ITERATION LOOP
C
      KOUNT=KOUNT+1
      IF(KOUNT.GT.40) GO TO 20
C
C     CALCULATE THE TEST FUNCTION OF BETA
C
```

```
      FB=GAMMA(1.+2./BETA)/(GAMMA(1.+1./BETA)**2
      DF=FB-A
      IF( ABS(DF).LT.1.0E-4.OR.DBETA.LT.I.E-3) GO TO 60
      IF(KOUNT.GT.I) GO TO I0
      IF(DF.GT.0.0) GO TO I2
      SIGN=-1.
      BETA=BETA/2.0
      GO TO 1
   12 CONTINUE
      SIGN=1.
      BETA=I.5*BETA
      GO TO 1
   10 CONTINUE
C
C     KOUNT IS GREATER THAN 1 AND ITERATION CONTINUES
C
      IF(DF.GT.0.0) GO TO 14
      IF(SIGN.GT.0.0) GO TO 5
C
C     CONTINUE HALVING OR DECREMENTING BETA
C
      IF(NREV.GT.0) GO TO 4
      BETA=BETA/2.0
      GO TO 40
    4 BETA=BETA-DBETA
      IF(BETA.LE.0.0) BETA=DBETA/2.0
   40 CONTINUE
      GO TO 1
    5 CONTINUE
C
C     SIGN REVERSAL HAS OCCURRED
C
      SIGN=-1.
      NREV=NREV+1
      IF(NREV.GT.I) DBETA=DBETA/2.0
      BETA=BETA-DBETA
      IF(BETA.LE.0.0) BETA=DBETA/2.0
      GO TO I
   I4 CONTINUE
C
C     DF IS POSITIVE; BETA MUST INCREASE.
C
      IF(SIGN.GT.0.0) GO TO 2
C
C     A SIGN REVERSAL HAS OCCURRED.
C
      SIGN=I.
      NREV=NREV+1
      IF(NREV.GT.1) DBETA=DBETA/2.0
      BETA=BETA+DBETA
      GO TO I
    2 CONTINUE
C
C     CONTINUE DOUBLING OR INCREMENTING BETA
C
      IF(NREV.GT.0) GO TO 45
      BETA=I.5*BETA
      GO TO 1
```

```
      45 CONTINUE
         BETA=BETA+DBETA
         GO TO 1
      20 CONTINUE
C
C        PRINT ERROR MESSAGE
C
         WRITE (6,50) BETA
      50 FORMAT(1H0,'ERROR.  COUNT LIMIT OF 40 EXCEEDED.  BETA = ',F10.4)
         CALL EXIT
         STOP
      60 CONTINUE
C
C        CONVERGENCE OF BETA ACHIEVED
C
         ETA=A1/GAMMA(1.+1./BETA)
         FN=FLOAT(N)
         SDBETA=SQRT(BETA*(2.0*BETA-1.)/FN)*(FN-2.)/(FN+2.)
         WRITE (6,69)
      69 FORMAT(1H0,'PARAMETER ESTIMATES OF THE C.D.F. OF DELP OBTAINED BY
        1MATCHING MOMENTS')
         WRITE (6,70) BETA,ETA,DF,DBETA,KOUNT,SDBETA
      70 FORMAT(1H0,'BETA = ',F10.4,'  ETA = ',F10.4,'    DF = ',F10.5,
        1 '    DBETA = ',F10.5,'    KOUNT = ',I3/1H0,
        2 'APPROXIMATE STD. DEV. OF ESTIMATE OF BETA = ',F10.5)
C
C        OBTAIN MAX LIKELIHOOD ESTIMATES
C
         LOOP=1
         ALPHA=BETA
         ALPHA1=BETA
      79 CONTINUE
         SUM1=0.
         SUM2=0.
         SUM3=0.
         SUM4=0.
         DO 80 I=1,N
         IF(T(I).LE.0.0) GO TO 100
         TIA=T(I)**ALPHA
         ALTI=ALOG(T(I))
         SUM1=SUM1+TIA
         SUM2=SUM2+TIA*ALTI
         SUM3=SUM3+ALTI
         SUM4=SUM4+TIA*ALTI**2
      80 CONTINUE
         HLAMB=FN/SUM1
         ALPHA=FN/(HLAMB*SUM2-SUM3)
         LOOP=LOOP+1
         DA=ABS(ALPHA-ALPHA1)
         ALPHA1=ALPHA
         IF(DA.LE.1.E-3.OR.LOOP.GE.9) GO TO 81
         GO TO 79
      81 CONTINUE
C
C        ALPHA CONVERGENCE ACHIEVED
C
C        CALCULATE CONFIDENCE INTERVALS FOR MAX LIKELIHOOD PARAM. ESTIMATES.
C        SEE P. 178, LLOYD AND LIPOW, RELIABILITY, C. 1962.
```

```
C
      PLL=-FN/HLAMB**2
      PAL=-SUM2
      PAA=-FN/ALPHA**2-HLAMB*SUM4
      DENOM=PAA*PLL-PAL**2
      VARA=-PLL/DENOM
      VARL=-PAA/DENOM
      COVAL=PAL/DENOM
C
C     CALCULATE LOWER AND UPPER 95% CONFIDENCE LIMITS
C
      TSTAT=STUDNT(0.025,N-2)
      SDA=SQRT(VARA)
      SDL=SQRT(VARL)
      RHOAL=COVAL/SDA/SDL
      CLLA=TSTAT*SDA
      CLUA=ALPHA+CLLA
      CLLA=ALPHA-CLLA
      CLLL=TSTAT*SDL
      CLUL=HLAMB+CLLL
      CLLL=HLAMB-CLLL
      BETA=ALPHA
      ETA=(1./HLAMB)**(1./BETA)
      WRITE (6,90) BETA,CLLA,CLUA,ETA,HLAMB,CLLL,CLUL
   90 FORMAT(1H0,'ESTIMATES BY MAXIMUM LIKELIHOOD'/1H0,'POINT ESTIMATES'
     1 ,25X,'95% CONF. INTERVAL'/1H ,4HBETA,11X,F10.5,10X,F10.5,5X,
     2 F10.5/1H ,7HETA,PSI,8X,F10.5/1H ,15HLAMBDA,PSI**-BE,F10.5,10X,
     3 F10.5,5X,F10.5)
      WRITE (6,91) SDA,SDL,RHOAL
   91 FORMAT(1H0,'STD. DEV. OF BETA =',F15.5/1H0,'STD. DEV. OF LAMBDA ='
     1 ,F15.5/1H0,'CORRELATION OF BETA AND LAMBDA =',F15.5)
      RETURN
  100 CONTINUE
      WRITE (6,102)
  102 FORMAT(1H0,'ZERO VALUES ENCOUNTERED PRECLUDES THE CALCULATION OF M
     1 AXIMUM LIKELIHOOD ESTIMATES.  MATCHING MOM. PARAMS. ARE KEPT.')
      RETURN
      ENTRY DISTF(X,CDFX)
C
C     TWO-PARAMETER WEIBULL MODEL
C
      CDFX=1.    - EXP(-(X/ETA)**BETA)
      RETURN
      END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  RANDMM  <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

      SUBROUTINE RANDMM (ISEED,X)
C
C    SUBROUTINE TO GENERATE A UNIFORM(0,1) DEVIATE X USING POSITIVE ODD
C         SEED ISEED.
C
      ISEED = ISEED * 65539
      IF (ISEED.LT.0)   ISEED = ISEED + 2147483647 + 1
      X = FLOAT(ISEED) * .4656613E-9
      RETURN
      END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  NORMXX  <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
```

```
443          SUBROUTINE NORMXX (AMIN,AMAX,AMEAN,SIGMA,X,ISEED)
       C
       C     SUBROUTINE TO GENERATE A NORMAL(AMEAN,SIGMA) DEVIATE LIMITED
       C          WITHIN (AMIN,AMAX) USING POSITIVE ODD SEED ISEED.
       C
444          CALL RANDMM (ISEED, Z)
445          CALL RANDMM (ISEED, X)
446          X = SIGMA * ( SQRT(-2.*ALOG(Z)) * COS(6.283185*X) ) + AMEAN
447          IF (X.LT.AMIN)    X = AMIN
448          IF (X.GT.AMAX)    X = AMAX
449          RETURN
450          END
       C     *************************  STUDNT  ******************************
451          FUNCTION STUDNT(ALPHA,NU)
       C
       C     THIS FUNCTION CALCULATES THE CRITICAL VALUE OF STUDENT'S T,
       C     GIVEN ONE OF THREE UPPER TAIL PROBS.--0.1, 0.05, 0.025--AND THE
       C     NUMBER OF DEGREES OF FREEDOM, NU.
       C     APPROXIMATION GIVEN IN AMS 55
       C
452          FN=NU
453          IF(ALPHA.GE.0.1) GO TO 1
454          IF(ALPHA.GE.0.05) GO TO 2
       C
       C     RESULT FOR ALPHA = 0.025
       C
455          X=1.960
456          GO TO 3
457        1 CONTINUE
       C
       C     RESULT FOR ALPHA = 0.1
       C
458          X=1.282
459          GO TO 3
460        2 CONTINUE
       C
       C     RESULT FOR ALPHA = 0.05
       C
461          X=1.645
462        3 CONTINUE
463          STUDNT=X+(X**3+X)/(4.*FN)+(5.*X**5+16.*X**3+3.*X)/(96.*FN*FN)
            1 +(3.*X**7+19.*X**5+17.*X**3-15.*X)/(384.*FN**3)
464          RETURN
465          END
       C     *************************  STUDNT  ******************************
       C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  GPPLOT  <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
466          SUBROUTINE GPPLOT (NLAYRS,XMIN,XMAX,YMIN,YMAX)
       C
       C--   GENERAL PURPOSE SUBROUTINE FOR PLOTS ON LINE PRINTER.  THE PLOT IS
       C     DISPLAYED ON A GRID 8" HIGH BY 10" WIDE WITH 1" DIVISIONS.  ONE- OR
       C     TWO-LAYER DISPLAYS ARE PROVIDED FOR--LAYER 1 IS STRICTLY ALPHAMER-
       C     IC, CONTAINS THE GRID, AND CAN BE USED FOR MULTIPLE FUNCTIONAL DIS-
       C     PLAYS OR SCATTER PLOTS WITHOUT FREQUENCY FEATURE; LAYER 2 IS NUMER-
       C     IC AND CAN BE USED FOR A SINGLE FUNCTIONAL DISPLAY OR FOR A SCATTER
       C     PLOT WITH FREQUENCY FEATURE.
       C
```

```
C     INPUTS TO GPPLOT, WHICH CLEARS AND SCALES THE PLOT ARRAY:
C          NLAYRS   NUMBER OF LAYERS IN THE PLOT ARRAY TO BE USED.
C          XMIN,XMAX,YMIN,YMAX
C                   LIMITS OF GRID SCALES.
C
      DIMENSION XLABEL(6), YLABEL(9)
      INTEGER ARRAY (51,101,2)
      DATA IBLANK,IHASH,IDASH / ' ','|','-' /
C
C     CLEAR LAYER 1, CREATE GRID, DEFINE LABELS
      DO I05 I=1,5I
      DO 105 J=I,I01
  I05 ARRAY(I,J,1) = IBLANK
      DRANGE = 0.125 * (YMAX-YMIN)
      DO I10 K=1,9
      I = 6*K - 4
      YLABEL(K) = YMIN + (K-1) * DRANGE
      DO I10 J=1,101
  I10 ARRAY(I,J,1) = IDASH
      DO 115 K=I,1I
      J = 10*K - 9
      DO 115 I=I,51
  115 ARRAY(I,J,1) = IHASH
      DRANGE = (XMAX-XMIN) / 5.0
      DO 120 K=I,6
  120 XLABEL(K) = XMIN + (K-1) * DRANGE
      X0 = XMIN
      RX = XMAX-XMIN
      Y0 = YMIN
      RY = YMAX-YMIN
      LAYERS = NLAYRS
      IF (LAYERS.EQ.I)   RETURN
C
C     CLEAR LAYER 2
      DO 155 I=I,51
      DO 155 J=I,101
  I55 ARRAY(I,J,2) = 0
      RETURN
C.......................................................................
C
      ENTRY GPLOD1 (X, Y, ICHAR)
C--  LOAD THE PLOT CHARACTER "ICHAR" AT THE (X,Y) POINT IN LAYER 1.
      LINE = ILINE(Y, Y0, RY)
      KOL = IKOLUM(X, X0, RX)
      ARRAY(LINE,KOL,1) = ICHAR
      RETURN
C.......................................................................
C
      ENTRY GPLINE(X,Y,X2,Y2,ICHAR)
C--  PLOT A "STRAIGHT LINE" USING "ICHAR" FROM (X,Y) TO (X2,Y2) IN LAYER
C    NUMBER I.
      LINE = ILINE(Y,Y0,RY)
      KOL = IKOLUM(X,X0,RX)
      LINE2 = ILINE(Y2,Y0,RY)
      KOL2 = IKOLUM(X2,X0,RX)
      ARRAY(LINE,KOL,1) = ICHAR
      ARRAY(LINE2,KOL2,1) = ICHAR
      LINES = LINE - LINE2
```

```
      KOLS = KOL - KOL2
      IF (LINES.LT.0)   LINES = -LINES
      IF (KOLS.LT.0)   KOLS = -KOLS
      IF (LINES.GT.KOLS)   GO TO 430
      IF (X2.EQ.X)   RETURN
      IF (KOL.LT.KOL2)   GO TO 400
      KI = KOL2 + I
      K2 = KOL - 1
      GO TO 410
  400 KI = KOL + I
      K2 = KOL2 - 1
  410 CONTINUE
      IF (K1.GT.K2)   RETURN
      DO 420 K=KI,K2
      XK = FLOAT(K)/100.*RX + X0
      YK = Y + (Y2-Y) * (XK-X) / (X2-X)
      LINE = ILINE(YK,Y0,RY)
  420 ARRAY(LINE,K,1) = ICHAR
      RETURN
  430 CONTINUE
      IF (LINE.LT.LINE2)   GO TO 440
      KI = LINE2 + 1
      K2 = LINE - I
      GO TO 450
  440 K1 = LINE + I
      K2 = LINE2 - 1
  450 CONTINUE
      IF (K1.GT.K2)   RETURN
      DO 460 K=KI,K2
      YK = FLOAT(50-K)/48.*RY + Y0
      XK = X + (X2-X) * (YK-Y) / (Y2-Y)
      KOL = IKOLUM(XK,X0,RX)
  460 ARRAY(K,KOL,1) = ICHAR
      RETURN
C.....................................................................
C
      ENTRY GPLOD2 (X, Y)
C--  ACCUMULATE AT THE (X,Y) POINT IN LAYER 2.
      LINE = ILINE(Y, Y0, RY)
      KOL = IKOLUM(X, X0, RX)
      ARRAY(LINE,KOL,2) = ARRAY(LINE,KOL,2) + 1
      RETURN
C.....................................................................
C
      ENTRY GPCONV (IVCHAR, NCHAR)
C--  CONVERT NUMERIC DATA IN LAYER 2 TO ALPHA FOR PRINTOUT
C        IVCHAR   VECTOR OF PLOT CHARACTERS TO BE USED IN LOADING LAYER
C                 NUMBER 2.
C        NCHAR  DIMENSION AND NUMBER OF CHARACTERS IN IVCHAR.
      DIMENSION IVCHAR(NCHAR)

      DO 225 I=1,51
      DO 225 J=I,101
      K = ARRAY(I,J,2)
      IF (K.GT.0)   GO TO 215
      ARRAY(I,J,2) = IBLANK
      GO TO 225
  215 CONTINUE
```

```
      IF (K.LE.NCHAR)   GO TO 220
      ARRAY(I,J,2) =IVCHAR(NCHAR)
      GO TO 225
  220 ARRAY(I,J,2) =IVCHAR(K)
  225 CONTINUE
      RETURN
C.............................................................................
C
      ENTRY GPPRNT
C--  PRINT THE PLOT
C    NOTE:  USER MUST PRINT HIS OWN IDENTIFYING LABELS BEFORE AND/OR
C    AFTER PRINTING THE PLOT, AS WELL AS ALLOWING FOR PROPER PAGING.
C    THE PLOT OCCUPIES 53 PRINT LINES.
C
    2 FORMAT(14X,101A1)
    3 FORMAT(1X,E11.4,2X,101A1)
    4 FORMAT('+',13X,101A1)
    5 FORMAT(T9,5(E11.4,9X),E11.4/)
C
      KOUNT1 = 5
      KOUNT2 = 9
      DO 310 I=1.51
      IF (KOUNT1.EQ.6)   GO TO 305
      WRITE (6,2) (ARRAY(1,J,1), J=1,101)
      IF (LAYERS.EQ.2)   WRITE (6,4) (ARRAY(I,J,2), J=1,101)
      KOUNT1 = KOUNT1 + 1
      GO TO 310
  305 WRITE (6,3) YLABEL(KOUNT2), (ARRAY(1,J,1), J=1,101)
      IF (LAYERS.EQ.2)   WRITE (6,4) (ARRAY(I,J,2), J=1,101)
      KOUNT1 = 1
      KOUNT2 = KOUNT2 - 1
  310 CONTINUE
      WRITE (6,5) XLABEL
      RETURN
      END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  ILINE   <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

      FUNCTION ILINE (YIN,YMIN,YRANGE)
C--  FUNCTION TO LOCATE PRINTER LINE CONTAINING PLOT POINT
      Z = (YIN-YMIN) / YRANGE
      ILINE = 50.5 - 48.0*Z
      IF (ILINE.LT.1)   ILINE = 1
      IF (ILINE.GT.51)   ILINE = 51
      RETURN
      END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  IKOLUM  <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

      FUNCTION IKOLUM (XIN. XMIN, XRANGE)
C--  FUNCTION TO LOCATE PRINTER COLUMN CONTAINING PLOT POINT
      Z = (XIN-XMIN) / XRANGE
      IKOLUM = 1.5 + 100.0*Z
      IF (IKOLUM.LT.1)   IKOLUM = 1
      IF (IKOLUM.GT.101)   IKOLUM = 101
      RETURN
      END
C>>>>>>>>>>>>>>>>>>>>>>>>>>>>>  PSCALE  <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

      SUBROUTINE PSCALE (ISCALE,IXY,DATMIN,DATMAX,XMIN,XMAX)
```

```
C
C     SUBROUTINE TO DEFINE PLOT SCALE LIMITS.  USED OPTIONALLY IN CONJUNC
C     TION WITH SUBROUTINE GPPLOT
C
C     INPUTS:
C          ISCALE   CONTROL PARAMETER:
C                   1  =>  USE GIVEN DATMIN AS XMIN; DEFINE XMAX ON BASIS
C                   OF DATMAX
C                   2  =>  USE GIVEN DATMAX AS XMAX; DEFINE XMIN ON BASIS
C                   OF DATMIN
C                   3  =>  DEFINE XMIN, XMAX ON BASIS OF DATMIN, DATMAX
C          IXY      CONTROL PARAMETER:
C                   1  =>  SCALING X-AXIS OF GPPLOT
C                   2  =>  SCALING Y-AXIS OF GPPLOT
C         DATMIN,DATMAX
C                   LIMITS OF DATA TO BE PLOTTED
C
C     OUTPUTS:
C         XMIN,XMAX
C                   PLOT SCALE LIMITS
C
       DIMENSION RMULT(4), MDIVS(2)
       DATA RMULT / 2., 5., 10., 20. /, MDIVS / 10, 8 /
C
     1 FORMAT(' ***PLOT SCALE ROUTINE FAILED.  RUN ABORTED.')
C
       NDIVS = MDIVS(IXY)
       DRANGE = (DATMAX-DATMIN) / NDIVS
       IEXPON = ALOG10 (DRANGE)
       IF (DRANGE.LT.1.0)   IEXPON = IEXPON - 1
       TENS = 10.0 ** IEXPON
       K = 1
       DO 100 I=1,2
       TEST = RMULT(I) * TENS
       IF (TEST.GE.DRANGE)   GO TO 110
   100 CONTINUE
       I = 3
   110 TEST = RMULT(I) * TENS
       GO TO (120,160,170), ISCALE
   120 XMIN = DATMIN
   130 XMAX = XMIN + NDIVS*TEST
       IF (XMAX.GE.DATMAX)   RETURN
       IF (K.EQ.1)   GO TO 150
   140 WRITE (6,1)
       STOP
   150 K = 2
       I = I + 1
       GO TO 110
   160 XMAX = DATMAX
       XMIN = XMAX - NDIVS*TEST
       IF (XMIN.LE.DATMIN)   RETURN
       IF (K-1)   150,150,140
   170 N = DATMIN/TEST
       IF (DATMIN.LT.0.0)   N = N - 1
       XMIN = N*TEST
       GO TO 130
       END
```

```
      $ENTRY
156         SUBROUTINE MLR(Y,X,NR,NC,NDATA,N,LZERO,PRINT,S,RSQ)
      C     IMPLICIT REAL *8 (A-H,O-Z)
      C
      C     Y=OATA MATRIX.  EACH OF THE FIRST N-1 COLUMNS REPRESENTS
      C     THE VECTOR OF VALUES OF ONE OF THE INOEPENDENT VARIABLES.
      C      THE N-TH COL. IS FOR THE DEPENDENT VARIABLE.
      C     CJ=SOLUTION VECTOR.  C(2) IS THE COEFFICIENT CORRESPONOING
      C     TO THE FIRST INOEPENOENT VARIABLE.
      C     C(1)=AVG. DEPENDENT VAR.
      C     NR=NUMBER OF ROWS IN STORAGE ALLOCATEO FOR THE OATA MATRIX.
      C     NC=NUMBER OF COLUMNS ALLOCATED IN THE OATA MATRIX STORAGE.
      C     NOATA=NUMBER OF VALUES OF EACH INDEPENOENT VARIABLE,
      C     IE, NUMBER OF OATA POINTS.
      C     N=NUMBER OF COLUMNS IN THE OATA MATRIX=TOTAL NO. OF VARS.
      C     IF LZERO IS ZERO, THE LEAOING (CONSTANT) COEFFICIENT IN EQ. WILL
      C        BE SET TO ZERO.
      C     PRINT=LOGICAL SWITCH SET TO .TRUE. FOR PRINT OF
      C     FITTEO FUNCTION ANO SET TO .FALSE. FOR NO PRINT.
      C     MEAN SQUARES=SUM OF SQUAREO OEVIATIONS FROM FITTEO FUNCTION
      C     OIVIDEO BY OEGREES OF FREEOOM, RETURNEO BY SUBROUTINE.
      C     RSQ IS THE R-SQUAREO STATISTIC RETURNEO BY SUBROUTINE.
      C
157         OIMENSION A(10,11)
158         DIMENSION X(NC),Y(NR,NC),B(10,11),XBAR(11),SIGMA(11)
159         REAL *8 SUM,SUMSQ,DABS,DSQRT
160         NA=10
161         NB=11
162         LOGICAL PRINT
```

```
      NPO=N+1
      NMO=N-1
      FN=NDATA
      DO 1 J=1,N
      SUM=0.
      DO 18 I=1,NDATA
   18 SUM=SUM+Y(I,J)
      XBAR(J)=SUM/FN
      SUMSQ=0.0
      DO 8 I=1,NDATA
    8 SUMSQ=SUMSQ+(Y(I,J)-XBAR(J))**2
    1 SIGMA(J)=DSQRT(DABS(SUMSQ)/(FN-1.0))
      LONE=0
      IF (LZERO.EQ.0) LONE=1
      ILOW=LONE+1
      DO 4 II=ILOW,N
      IM=II-1
      IF (IM.NE.0) GO TO 2
      A(1,1)=FN
      DO 3 J=2,NPO
    3 A(1,J)=0.0
      GO TO 4
    2 I=II-LONE
      DO 5 JJ=II,NPO
      J=JJ-LONE
      JM=JJ-1
      A(I,J)=0.0
      DO 5 K=1,NDATA
    5 A(I,J)=A(I,J)+(Y(K,IM)-XBAR(IM))/SIGMA(IM)
     .                  *(Y(K,JM)-XBAR(JM))/SIGMA(JM)
    4 CONTINUE
      N1=N-LONE
      N2=N1+1
      DO 6 I=2,N1
      K=I-1
      DO 6 J=1,K
    6 A(I,J)=A(J,I)
      DO 7 I=1,N1
      DO 7 J=1,N2
    7 B(I,J)=A(I,J)
      CALL AXBSOL(B,X,N1,NA,NB,1.0E-10*SIGMA(N))
      IF (.NOT.PRINT) GO TO 9
      PRINT 10
   10 FORMAT (1H04X1HI4X11HNORM. COEF.8X7HXBAR(I)7X8HSIGMA(I)2X
     . 13HUNIT RESIDUAL)

      DO 11 I=1,N1
      R=0.
      DO 12 J=1,N1
```

```
   12 R=R+A(I,J)*X(J)                                          00007700
      IF (A(I,N2).EQ.0.0) GO TO 11                              00007800
      R=(R-A(I,N2))/A(I,N2)                                     00007900
   11 PRINT 16,I,X(I),XBAR(I),SIGMA(I),R                        00008000
   16 FORMAT (1H I5,1P4E15.6)                                   00008100

    9 IF (LZERO.NE.0) GO TO 21                                  00008200
      DO 20 J=1,N1                                              00008300
   20 X(NPO-J)=X(N2-J)                                          00008400
      X(1)=0.0                                                  00008500
   21 SUM=X(1)                                                  00008600
      DO 19 J=1,NMO                                             00008700
      J2=J+1                                                    00008800
      SUM=SUM-X(J2)*XBAR(J)/SIGMA(J)                            00008900
   19 X(J2)=X(J2)*SIGMA(N)/SIGMA(J)                             00009000
      IF (LZERO.NE.0) X(1)=SIGMA(N)*SUM+XBAR(N)                 00009100
      IF (PRINT) PRINT 22,(I,X(I),I=1,N)                        00009200
   22 FORMAT ('0DIMENSIONALIZED COEFFICIENTS'/(' X('I2,')='1PE15.8))   00009300

      S=0.                                                      00009400
      DO 13 K=1,NDATA                                           00009500
      YC=X(1)                                                   00009600
      DO 14 I=2,N                                               00009700
   14 YC=YC+Y(K,I-1)*X(I)                                       00009800
      RES=YC-Y(K,N)                                             00009900
   13 S=S+RES**2                                                00010000
      RSQ=(SUMSQ-S)/SUMSQ                                       00010100
      DF=NDATA-N                                                00010200
      IF (LZERO.EQ.0) DF=DF+1.0                                 00010300
      IF (DF.LE.0.0) RETURN                                     00010400
      S=S/DF                                                    00010500
      IF (PRINT) PRINT 15,S,RSQ                                 00010600
   15 FORMAT (1H 5X9HRES. M.S.5X9HR-SQUARED/1H 1PE15.6,0PF15.6) 00010700


      RETURN                                                    00010800
      END                                                       00010900

      SUBROUTINE AXBSOL(A,X,N,NA,NB,EPSIL)                      00011000
C     IMPLICIT REAL *8 (A-H,O-Z)                                00011100
      DIMENSION A(NA,NB),X(N)                                   00011200
      NPO=N+1                                                   00011300
      NMO=N-1                                                   00011400
C     SET MATRIX SIZE COUNTER                                   00011500
      DO 2 J=1,NMO                                              00011600
      B=EPSIL                                                   00011700
      JROW=0                                                    00011800
      D=A(J,J)                                                  00011900
C     LOOK FOR LARGEST PIVOTAL ELEMENT OR ZERO ROW              00012000
      DO 3 I=J,N                                                00012100
C     IF (DABS(A(I,J))-DABS(B))3,3,4                            00012200
      IF ( ABS(A(I,J))- ABS(B))3,3,4                            00012300
    4 JROW=I                                                    00012400
      B=A(I,J)                                                  00012500
```

```
      O=B
    3 CONTINUE
C     IF JROW STILL ZERO, MINOR IS ZERO. IF GTR THAN J, SWITCH ROWS.
      IF (JROW-J)5,6,7
    7 DO 8 L=J,NPO
      C=A(J,L)
      A(J,L)=A(JROW,L)
    8 A(JROW,L)=C
C     DIVIDE ROW J BY PIVOTAL ELEMENT
    6 DO 9 L=J,NPO
    9 A(J,L)=A(J,L)/D
C     GET ZEROS INTO ALL ROWS OF COLUMN J BELOW PIVOT
      DO 2 IK=J,NMO
      B=A(IK+1,J)
      DO 2 JK=J,NPO
    2 A(IK+1,JK)=A(IK+1,JK)-B*A(J,JK)
C     GET SOLUTION VECTOR BY BACK-SUBSTITUTION
      X(N)=A(N,NPO)/A(N,N)
      DO 10 IK=1,NMO
      I=N-IK
      X(I)=A(I,NPO)
      DO 10 JK=1,IK
      J=NPO-JK
   10 X(I)=X(I)-X(J)*A(I,J)
      RETURN
    5 PRINT 15,J,J
   15 FORMAT (20H0MINOR OF COFACTOR (I1,1H,I1,9H) IS ZERO)

      CALL EXIT
      END
```

```
TEST DATA FOR 8 IN HOW, M188E1 PROP CHG--SECURED CARGO (COLD)  SET C1

SAMPLE =  27  F-S CRITICAL VALUE =      2.485     F-S LEVEL OF RISK =     0.0500     DEG. OF POLY. REGRES. =  1

   AVG PMAX,KSI   AVG DELP,PSI   AVG TMDEL,MS
       37.5962          513.3          145.4

  SD:  PMAX,KSI       DELP,PSI       DELAY,MS
         0.7719       490.8469        44.9309

CORRELATION MATRIX:

           PMAX           DELP          DELAY
         1.0000         0.0456         0.0134
         0.0456         1.0000        -0.1956
         0.0134        -0.1956         1.0000

ANALYSIS BASED ON SIMPLE LINEAR REGRESSION OF PMAX ON DELP:

SLOPE =    0.00007 KSI/PSI     STD. ERR. OF EST. OF PMAX GIVEN DELP =   0.78632 KSI

STD. DEV. OF SLOPE =   0.00031     95% CONF. LIMITS:    -0.00058   0.00072     T-STATISTIC (CRITICAL) =   2.05957

PARTIAL CORRELATION COEFS. WITH DEPENDENCE OF PMAX ON DELP AND DELAY ASSUMED

 DELP, GIVEN DT DT, GIVEN DELP
         0.0492         0.0227

FRACTION OF VARIANCE DUE TO DELP =          0.0024

FRACTION OF VARIANCE DUE TO DELAY =          0.0005

    I    NORM. COEF.       XBAR(I)       SIGMA(I)  UNIT RESIDUAL
    1  -0.000000E-01   5.132590E 02   4.908474E 02   0.000000E-01
    2   4.561469E-02   3.759628E 01   7.718509E-01  -8.041248E-07

DIMENSIONALIZED COEFFICIENTS
X( 1)= 3.75594600E 01
X( 2)= 7.17284700E-05
     RES. M.S.      R-SQUARED
   6.182953E-01       0.002080

REGRESSION COEFS. FOR PMAX ON DELP:

         ZEROTH          FIRST            2ND            3RD
     0.37559E 02    0.71728E-04

FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION:        0.00208
```

DISTRIBUTION OF RESIDUALS FROM PMAX-DELP REGRESSION

INDEX	RESID,KSI	MED RANK	NORM D.F.	DSTAR	SSTAR
1	-1.7818	0.0357	0.0104	0.0266	0.0266
2	-1.6850	0.0714	0.0144	0.0596	0.0863
3	-1.1924	0.1071	0.0610	0.0501	0.1364
4	-1.0674	0.1429	0.0831	0.0650	0.2014
5	-0.7050	0.1786	0.1803	0.0321	0.2335
6	-0.5801	0.2143	0.2259	0.0407	0.2742
7	-0.5765	0.2500	0.2273	0.0320	0.3062
8	-0.1681	0.2857	0.4137	0.1545	0.4606
9	-0.0990	0.3214	0.4489	0.1526	0.6133
10	-0.0861	0.3571	0.4555	0.1222	0.7355
11	0.0108	0.3929	0.5056	0.1352	0.8707
12	0.0123	0.4286	0.5063	0.0989	0.9696
13	0.0342	0.4643	0.5177	0.0732	1.0429
14	0.0405	0.5000	0.5210	0.0395	1.0823
15	0.1405	0.5357	0.5723	0.0538	1.1361
16	0.3927	0.5714	0.6947	0.1392	1.2753
17	0.3943	0.6071	0.6955	0.1029	1.3782
18	0.4040	0.6429	0.6999	0.0702	1.4484
19	0.5210	0.6786	0.7504	0.0837	1.5321
20	0.5346	0.7143	0.7559	0.0522	1.5843
21	0.5481	0.7500	0.7614	0.0207	1.6050
22	0.5694	0.7857	0.7699	0.0449	1.6500
23	0.6299	0.8214	0.7930	0.0588	1.7088
24	0.8048	0.8571	0.8517	0.0372	1.7460
25	0.8141	0.8929	0.8545	0.0715	1.8175
26	0.8207	0.9286	0.8564	0.1065	1.9240
27	1.2697	0.9643	0.9502	0.0498	1.9738

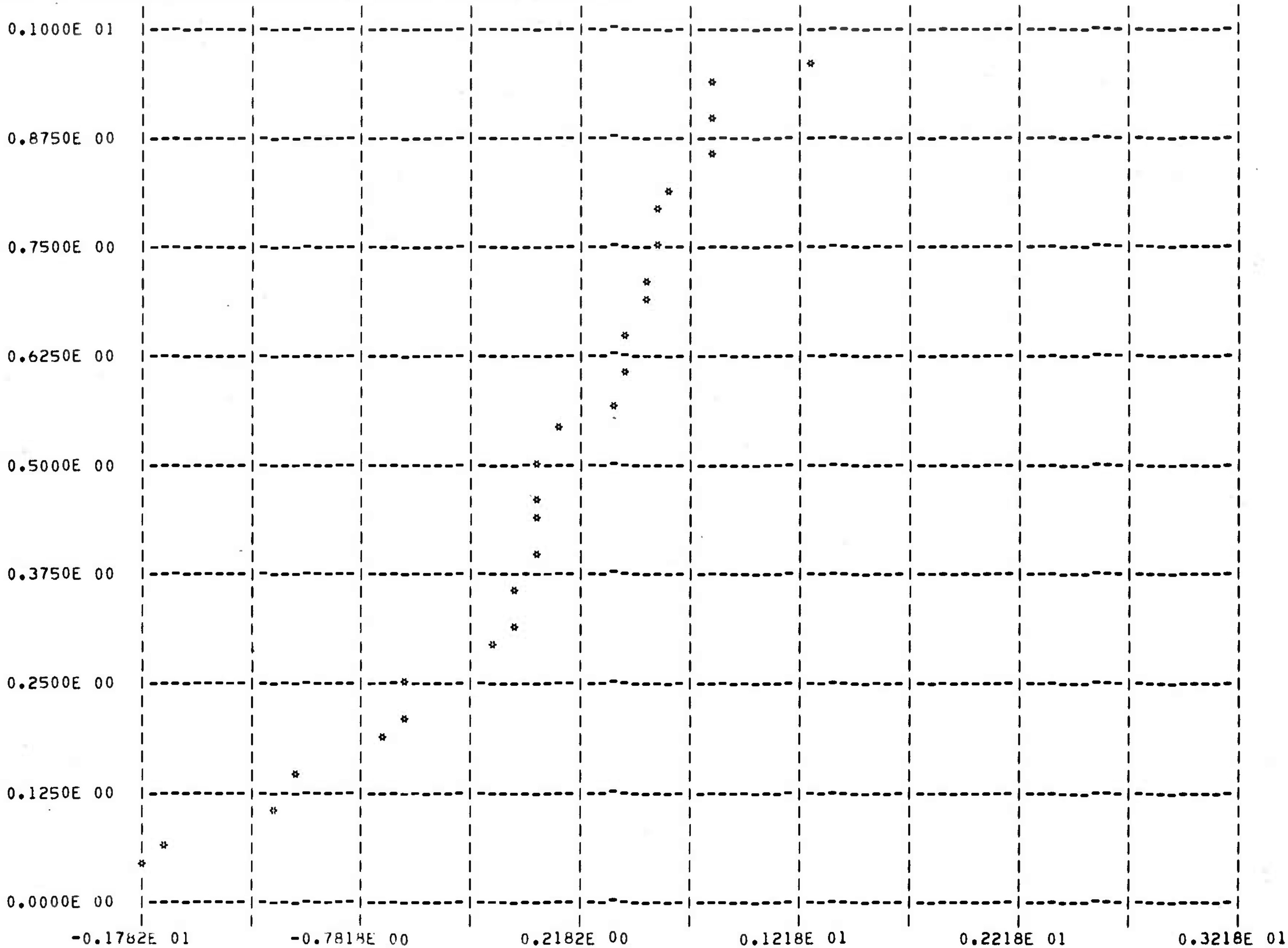
RESIDUALS ARE PLAUSIBLY GAUSSIAN.

SSTAR IN F-S TEST, 1.9738 IS LESS THAN THE CRITICAL VALUE, 2.4854

LILLIEFORS (K-S) TEST SHOWS RESIDUALS ARE GAUSSIAN.
TEST STATISTIC 0.1545 IS LESS THAN THE CRITICAL VALUE 0.1665 WITH A RISK OF 0.0500

MEAN AND STD. DEV. OF RESIDUALS (KSI): 0.00002 0.77105

PLOT OF CUMULATIVE DISTRIBUTION FUNCTION OF PMAX RESIDUALS



I	NORM. COEF.	XBAR(I)	SIGMA(I)	UNIT RESIDUAL
1	-0.000000E-01	1.454074E 02	4.493092E 01	0.000000E-01
2	1.335169E-02	3.759628E 01	7.718509E-01	-1.717001E-07

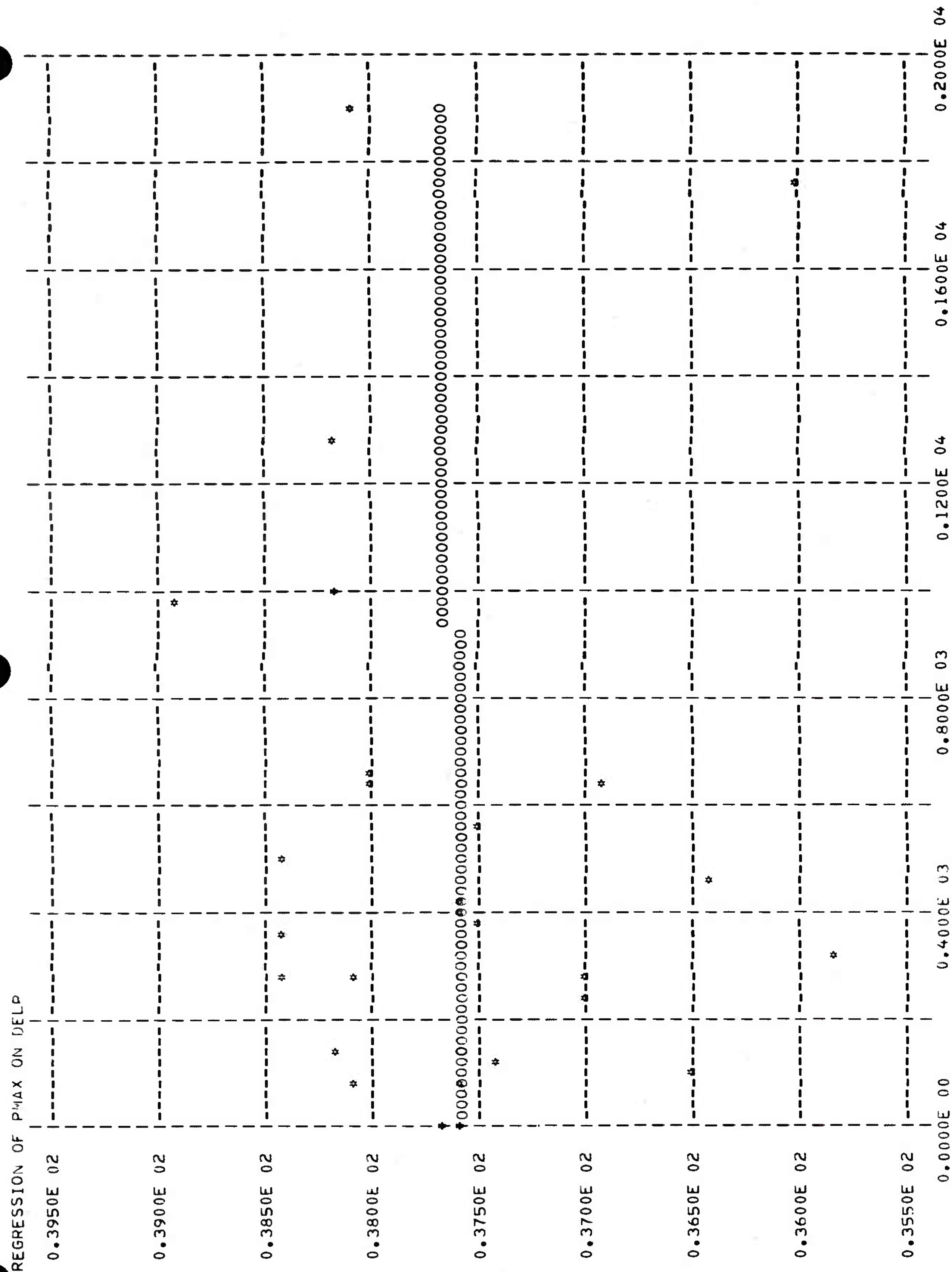
```
DIMENSIONALIZED COEFFICIENTS
X( 1)= 3.75629200E 01
X( 2)= 2.29363500E-04
     RES. M.S.        R-SQUARED
   6.194732E-01         0.000179
```

REGRESSION COEFS. FOR PMAX ON DELAY:

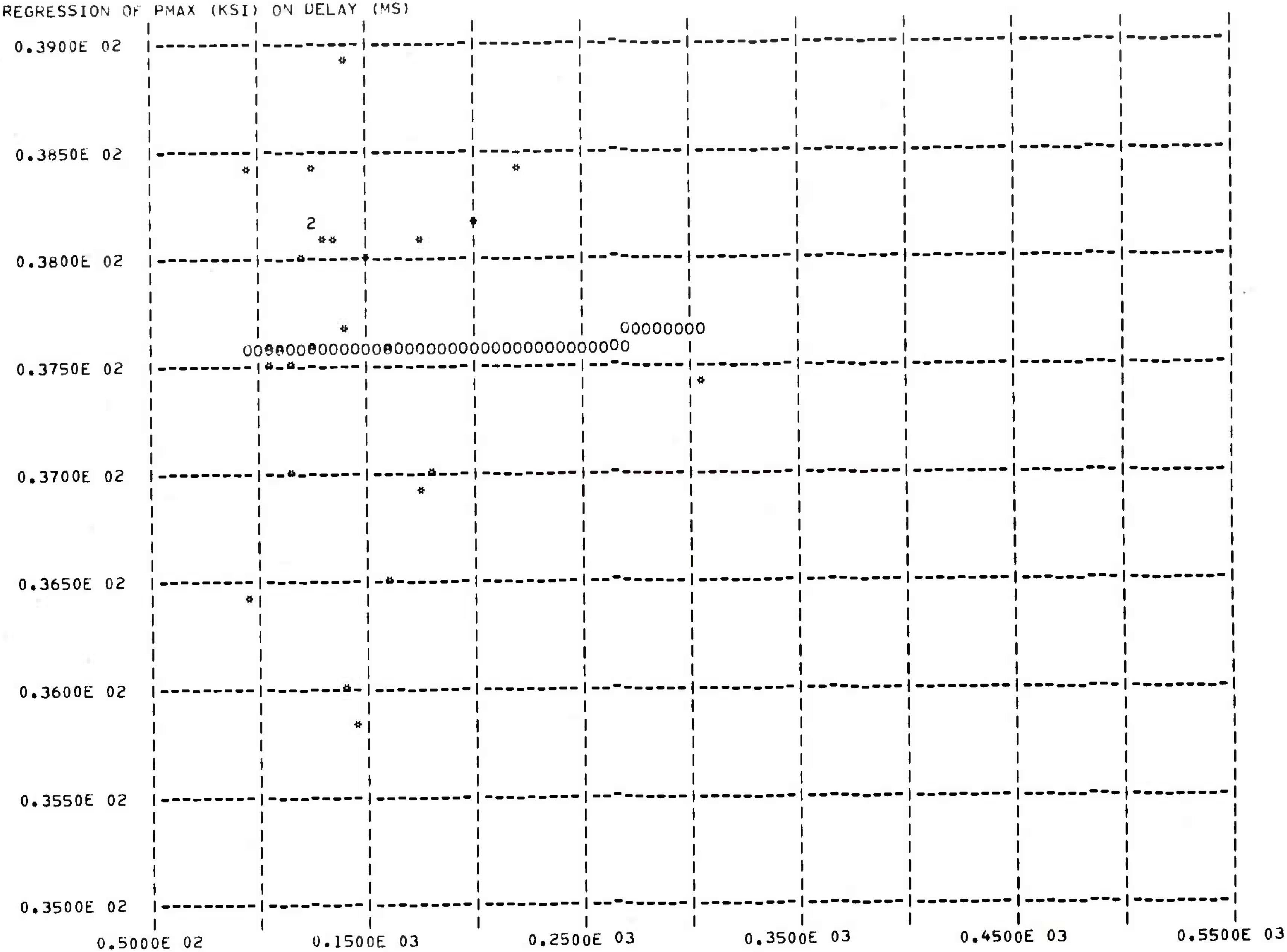
ZEROTH	FIRST	2ND	3RD
0.37563E 02	0.22936E-03		

FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION: 0.00018

INDEX	MED RANK	PMAX,KSI	DELP,PSI	DELAY,MS
1	0.0357	35.8000	0.00	93.00
2	0.0714	36.0000	0.00	95.00
3	0.1071	36.4000	83.00	105.00
4	0.1429	36.5000	89.00	106.00
5	0.1786	36.9000	110.00	108.00
6	0.2143	37.0000	120.00	113.00
7	0.2500	37.0000	148.00	114.00
8	0.2857	37.4000	238.00	120.00
9	0.3214	37.5000	272.00	125.00
10	0.3571	37.5000	276.00	126.00
11	0.3929	37.6000	288.00	126.00
12	0.4286	37.6000	312.00	127.00
13	0.4643	37.6000	369.00	130.00
14	0.5000	37.6000	371.00	135.00
15	0.5357	37.7000	394.00	142.00
16	0.5714	38.0000	414.00	142.00
17	0.6071	38.0000	459.00	142.00
18	0.6429	38.1000	498.00	145.00
19	0.6786	38.1000	551.00	150.00
20	0.7143	38.1000	635.00	160.00
21	0.7500	38.2000	644.00	162.00
22	0.7857	38.2000	667.00	176.00
23	0.8214	38.2000	987.00	177.00
24	0.8571	38.4000	992.00	178.00
25	0.8929	38.4000	1288.00	202.00
26	0.9286	38.4000	1750.00	222.00
27	0.9643	38.9000	1903.00	305.00


REGRESSION OF PMAX ON DELP
0.3950E 02
0.3900E 02
0.3850E 02
0.3800E 02
0.3750E 02
0.3700E 02
0.3650E 02
0.3600E 02
0.3550E 02
0.0000E 00
0.4000E 03
0.8000E 03
0.1200E 04
0.1600E 04
0.2000E 04

REGRESSION OF PMAX (KSI) ON DELAY (MS)



PARAMETER ESTIMATES OF THE C.D.F. OF DELP OBTAINED BY MATCHING MOMENTS

BETA = 1.0461 ETA = 522.5786 DF = -0.00023 DBETA = 0.00078 KOUNT = 15

APPROXIMATE STD. DEV. OF ESTIMATE OF BETA = 0.17733

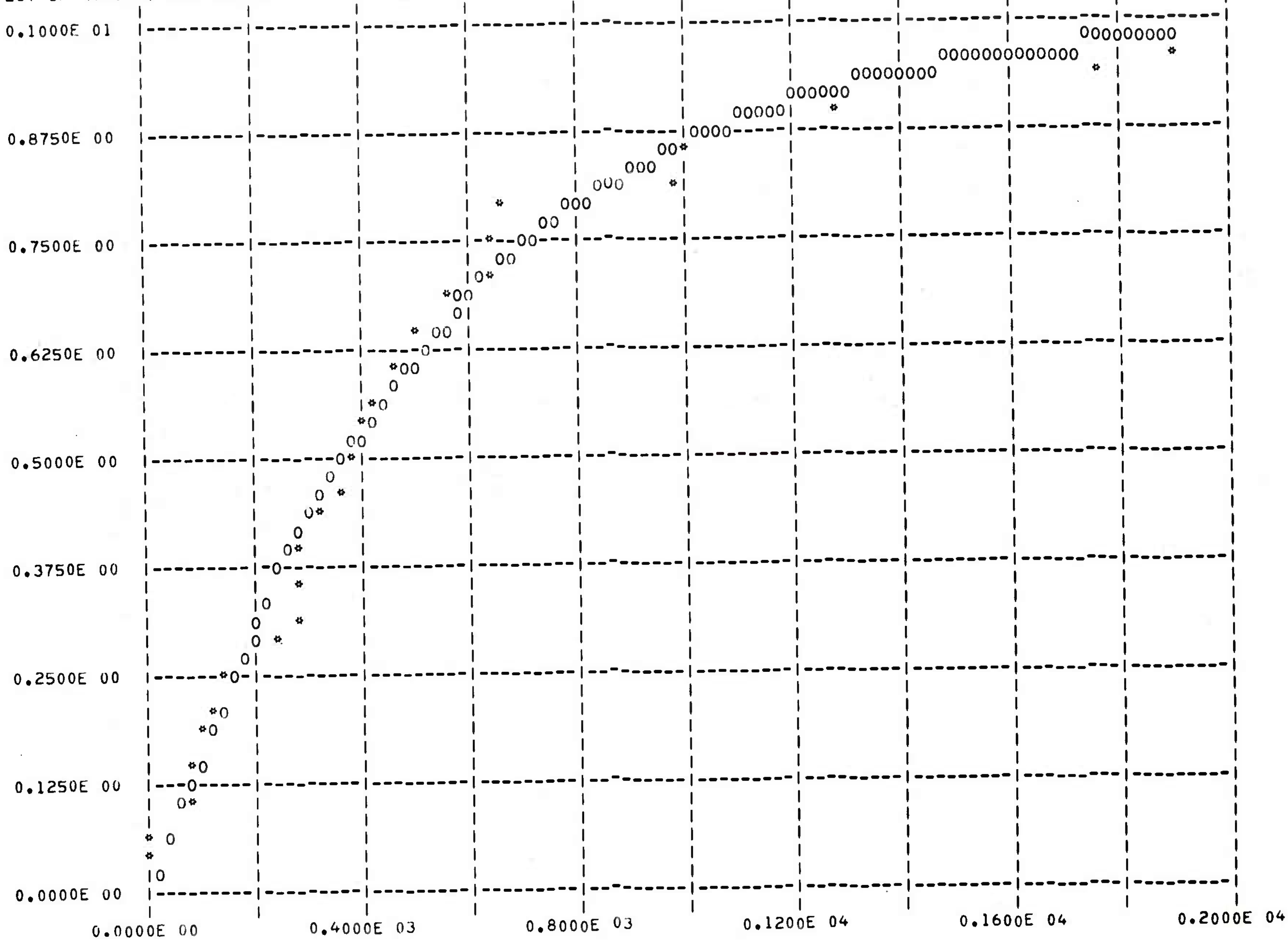
ZERO VALUES ENCOUNTERED PRECLUDES THE CALCULATION OF M AXIMUM LIKELIHOOD ESTIMATES. MATCHING MOM. PARAMS. ARE KEPT.

INDEX	DELP,KSI	CDF	DSTAR	SSTAR
1	0.0000	-0.0000	0.0370	0.0370
2	0.0000	-0.0000	0.0741	0.1111
3	83.0000	0.1358	0.0617	0.1728
4	89.0000	0.1453	0.0342	0.2070
5	110.0000	0.1779	0.0298	0.2367
6	120.0000	0.1931	0.0291	0.2658
7	148.0000	0.2345	0.0248	0.2906
8	238.0000	0.3555	0.0962	0.3868
9	272.0000	0.3965	0.1002	0.4870
10	276.0000	0.4012	0.0679	0.5549
11	288.0000	0.4150	0.0447	0.5996
12	312.0000	0.4418	0.0344	0.6339
13	369.0000	0.5009	0.0564	0.6904
14	371.0000	0.5028	0.0213	0.7117
15	394.0000	0.5249	0.0307	0.7424
16	414.0000	0.5433	0.0493	0.7917
17	459.0000	0.5823	0.0473	0.8389
18	498.0000	0.6136	0.0531	0.8920
19	551.0000	0.6525	0.0512	0.9432
20	635.0000	0.7066	0.0342	0.9774
21	644.0000	0.7118	0.0659	1.0433
22	667.0000	0.7249	0.0899	1.1332
23	987.0000	0.8570	0.0422	1.1754
24	992.0000	0.8585	0.0304	1.2058
25	1288.0000	0.9234	0.0345	1.2403
26	1750.0000	0.9710	0.0451	1.2854
27	1903.0000	0.9790	0.0210	1.3064

SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F. USING THE F-S TEST S STAR = 1.3064

LILLIEFORS (K-S) TEST SHOWS SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F.
TEST STATISTIC 0.1002 IS LESS THAN THE CRITICAL VALUE 0.1665 WITH A RISK OF 0.0500

PLOT OF HYPOTH. AND SAMPLE C.O.F.S OF OELP



TEST DATA FOR 8 IN. HOW.,M188E1 CHG.,MERGE OF SETS C2+C3,SEQ. ROUGH HAND.(COLD)

SAMPLE = 27 F-S CRITICAL VALUE = 2.485 F-S LEVEL OF RISK = 0.0500 DEG. OF POLY. REGRES. = 3

AVG PMAX,KSI	AVG DELP,PSI	AVG TMDEL,MS
37.0851	675.3	166.8

SD: PMAX,KSI	DELP,PSI	DELAY,MS
0.8416	1136.6830	65.4170

CORRELATION MATRIX:

PMAX	DELP	DELAY
1.0000	0.6697	-0.0943
0.6697	1.0000	0.1489
-0.0943	0.1489	1.0000

ANALYSIS BASED ON SIMPLE LINEAR REGRESSION OF PMAX ON DELP:

SLOPE = 0.00050 KSI/PSI STD. ERR. OF EST. OF PMAX GIVEN DELP = 0.63732 KSI

STD. DEV. OF SLOPE = 0.00011 95% CONF. LIMITS: 0.00027 0.00072 T-STATISTIC (CRITICAL) = 2.05957

PARTIAL CORRELATION COEFS. WITH DEPENDENCE OF PMAX ON DELP AND DELAY ASSUMED

DELP, GIVEN DT	DT, GIVEN DELP
0.6946	-0.2642

FRACTION OF VARIANCE DUE TO DELP = 0.4824

FRACTION OF VARIANCE DUE TO DELAY = 0.0698

I	NORM. COEF.	XBAR(I)	SIGMA(I)	UNIT RESIDUAL
1	-0.000000E-01	6.752590E 02	1.136687E 03	0.000000E-01
2	-2.945518E-01	1.700179E 06	5.773493E 06	-8.762936E-07
3	2.865553E 00	7.168160E 09	3.038551E 10	-1.786141E-06
4	-1.947622E 00	3.708517E 01	8.415649E-01	-9.438726E-07

DIMENSIONALIZED COEFFICIENTS
X(1)= 3.69089300E 01
X(2)=-2.18076200E-04
X(3)= 4.17693100E-07
X(4)=-5.39418300E-11

RES. M.S.	R-SQUARED
4.204599E-01	0.474825

REGRESSION COEFS. FOR PMAX ON DELP:

ZEROTH	FIRST	2ND	3RD
0.36909E 02	-0.21808E-03	0.41769E-06	-0.53942E-10

FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION: 0.47483

DISTRIBUTION OF RESIDUALS FROM PMAX-DELP REGRESSION

INDEX	RESID,KSI	MED RANK	NORM D.F.	DSTAR	SSTAR
1	-1.0011	0.0357	0.0504	0.0504	0.0504
2	-0.7822	0.0714	0.0998	0.0628	0.1131
3	-0.7624	0.1071	0.1056	0.0315	0.1447
4	-0.7501	0.1429	0.1094	0.0388	0.1835
5	-0.7075	0.1786	0.1230	0.0622	0.2457
6	-0.5861	0.2143	0.1683	0.0540	0.2996
7	-0.5797	0.2500	0.1709	0.0883	0.3880
8	-0.3273	0.2857	0.2957	0.0365	0.4244
9	-0.3089	0.3214	0.3062	0.0271	0.4515
10	-0.1089	0.3571	0.4291	0.0958	0.5473
11	-0.0939	0.3929	0.4388	0.0684	0.6157
12	-0.0833	0.4286	0.4456	0.0382	0.6540
13	-0.0224	0.4643	0.4853	0.0409	0.6948
14	0.0744	0.5000	0.5486	0.0671	0.7619
15	0.0867	0.5357	0.5565	0.0380	0.7999
16	0.0972	0.5714	0.5633	0.0293	0.8292
17	0.1085	0.6071	0.5706	0.0590	0.8882
18	0.1121	0.6429	0.5729	0.0937	0.9820
19	0.1478	0.6786	0.5957	0.1080	1.0899
20	0.2911	0.7143	0.6834	0.0573	1.1473
21	0.3095	0.7500	0.6941	0.0837	1.2309
22	0.3118	0.7857	0.6954	0.1194	1.3503
23	0.4538	0.8214	0.7716	0.0803	1.4306
24	0.5182	0.8571	0.8023	0.0866	1.5173
25	0.7717	0.8929	0.8971	0.0288	1.5461
26	1.2204	0.9286	0.9773	0.0514	1.5975
27	1.6112	0.9643	0.9959	0.0329	1.6304

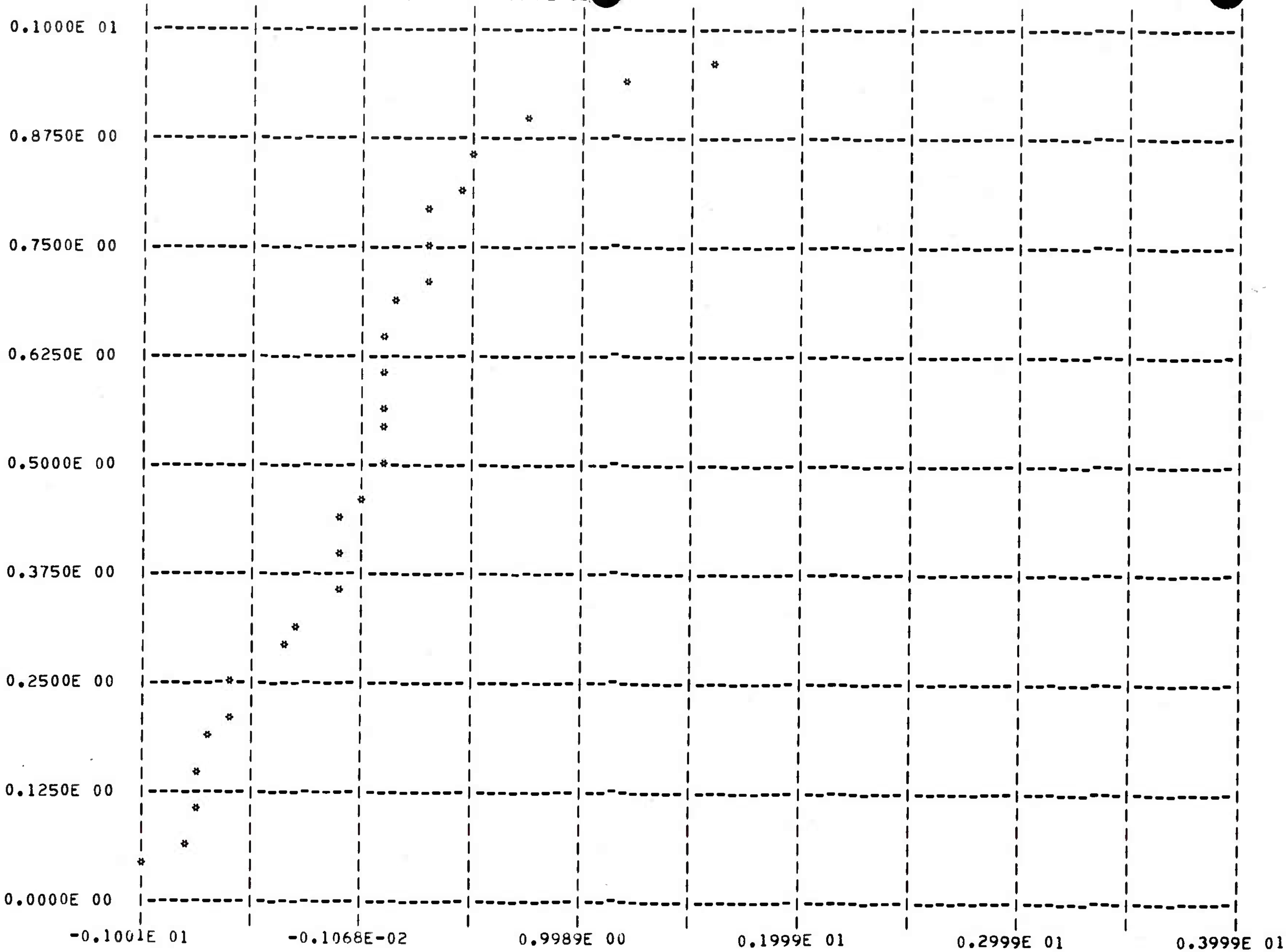
RESIDUALS ARE PLAUSIBLY GAUSSIAN.

SSTAR IN F-S TEST, 1.6304 IS LESS THAN THE CRITICAL VALUE, 2.4854

LILLIEFORS (K-S) TEST SHOWS RESIDUALS ARE GAUSSIAN.
TEST STATISTIC 0.1194 IS LESS THAN THE CRITICAL VALUE 0.1665 WITH A RISK OF 0.0500

MEAN AND STD. DEV. OF RESIDUALS (KSI): 0.00002 0.60987

PLOT OF CUMULATIVE DISTRIBUTION FUNCTION OF PMAX RESIDUALS



I	NORM. COEF.	XBAR(I)	SIGMA(I)	UNIT RESIDUAL
1	-0.000000E-01	1.667778E 02	6.541719E 01	0.000000E-01
2	-6.137496E 00	3.193574E 04	2.934265E 04	8.751107E-05

3	1.220910E 01	7.138332E 06	1.119845E 07	9.293784E-05
4	-6.349072E 00	3.708517E 01	8.415649E-01	1.207029E-04

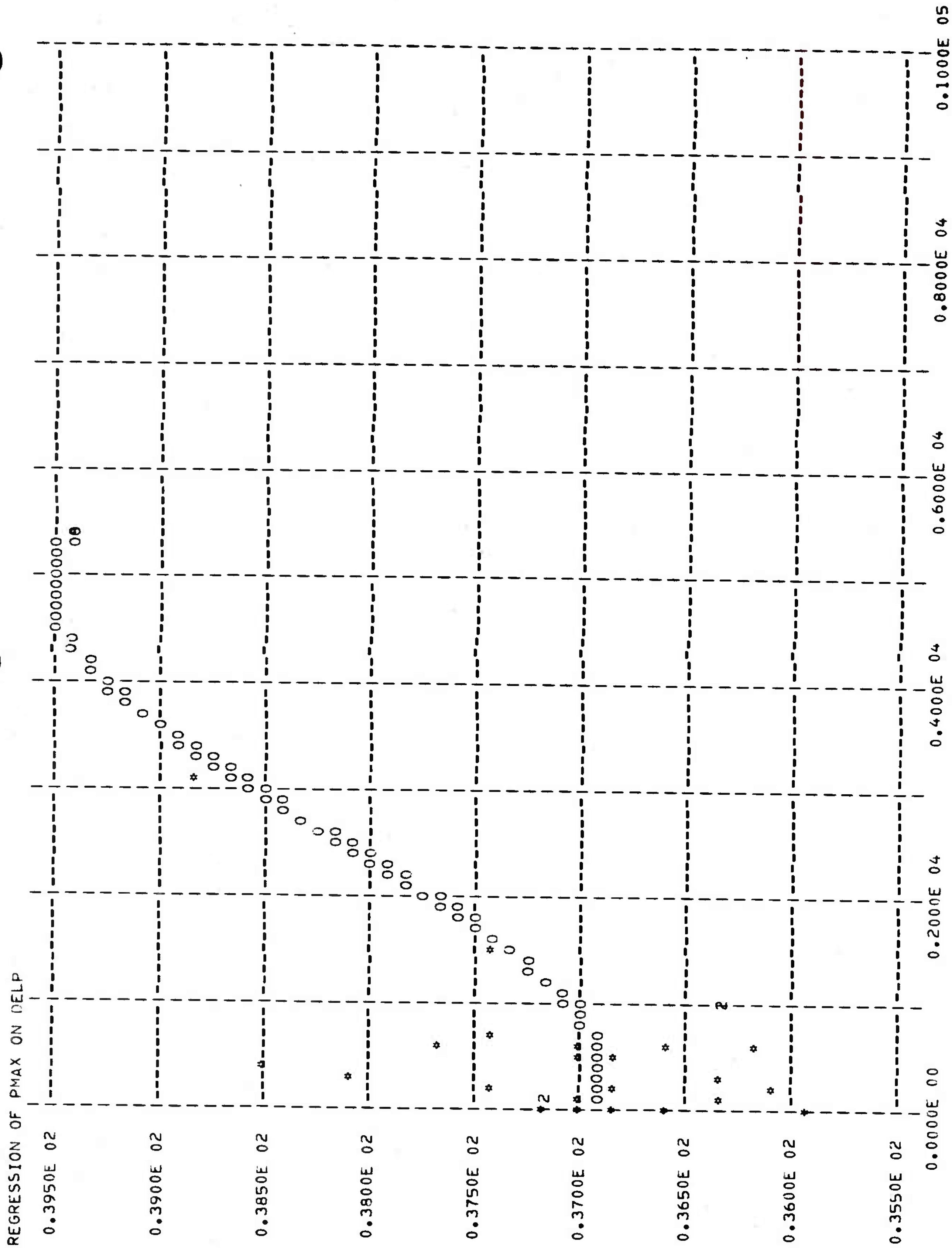
```
DIMENSIONALIZED COEFFICIENTS
X( 1)= 4.24765100E 01
X( 2)=-7.89563000E-02
X( 3)= 3.50164200E-04
X( 4)=-4.77133400E-07
     RES. M.S.       R-SQUARED
   7.337421E-01         0.083521
```

REGRESSION COEFS. FOR PMAX ON DELAY:

ZEROTH	FIRST	2ND	3RD
0.42477E 02	-0.78956E-01	0.35016E-03	-0.47713E-06

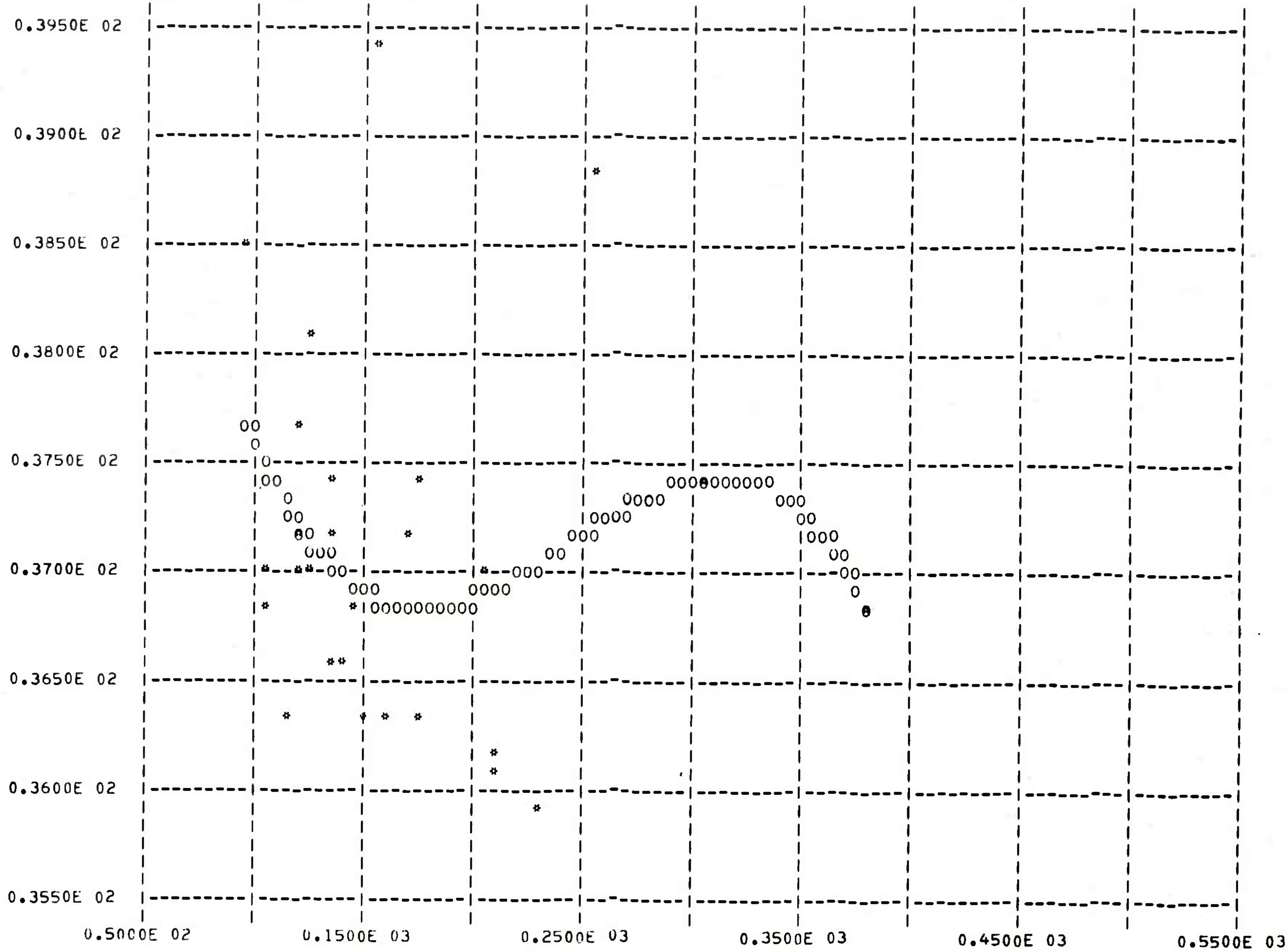
FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION: 0.08352

INDEX	MED RANK	PMAX,KSI	DELP,PSI	DELAY,MS
1	0.0357	35.9000	0.00	93.00
2	0.0714	36.1000	0.00	105.00
3	0.1071	36.2000	0.00	105.00
4	0.1429	36.3000	30.00	117.00
5	0.1786	36.3000	39.00	120.00
6	0.2143	36.3000	106.00	120.00
7	0.2500	36.3000	124.00	121.00
8	0.2857	36.6000	127.00	125.00
9	0.3214	36.6000	143.00	127.00
10	0.3571	36.8000	174.00	135.00
11	0.3929	36.8000	190.00	136.00
12	0.4286	36.8000	197.00	137.00
13	0.4643	37.0000	296.00	140.00
14	0.5000	37.0000	302.00	143.00
15	0.5357	37.0000	436.00	150.00
16	0.5714	37.0000	458.00	154.00
17	0.6071	37.2000	476.00	160.00
18	0.6429	37.2000	556.00	171.00
19	0.6786	37.2000	584.00	173.00
20	0.7143	37.4000	644.00	175.00
21	0.7500	37.4000	648.00	205.00
22	0.7857	37.4000	713.00	211.00
23	0.8214	37.7000	990.00	212.00
24	0.8571	38.1000	1017.00	232.00
25	0.8929	38.5000	1479.00	253.00
26	0.9286	38.8000	3115.00	303.00
27	0.9643	39.4000	5388.00	380.00



REGRESSION OF PMAX ON DELP
0.3950E 02
0.3900E 02
0.3850E 02
0.3800E 02
0.3750E 02
0.3700E 02
0.3650E 02
0.3600E 02
0.3550E 02
0.0000E 00
0.2000E 04
0.4000E 04
0.6000E 04
0.8000E 04
0.1000E 05

REGRESSION OF PMAX (KSI) ON DELAY (MS)



PARAMETER ESTIMATES OF THE C.D.F. OF DELP OBTAINED BY MATCHING MOMENTS

A = 0.6211 ETA = 468.7659 DF = 0.00466 ETA = 0.00078 KOUNT = 16

APPROXIMATE STD. DEV. OF ESTIMATE OF BETA = 0.06434

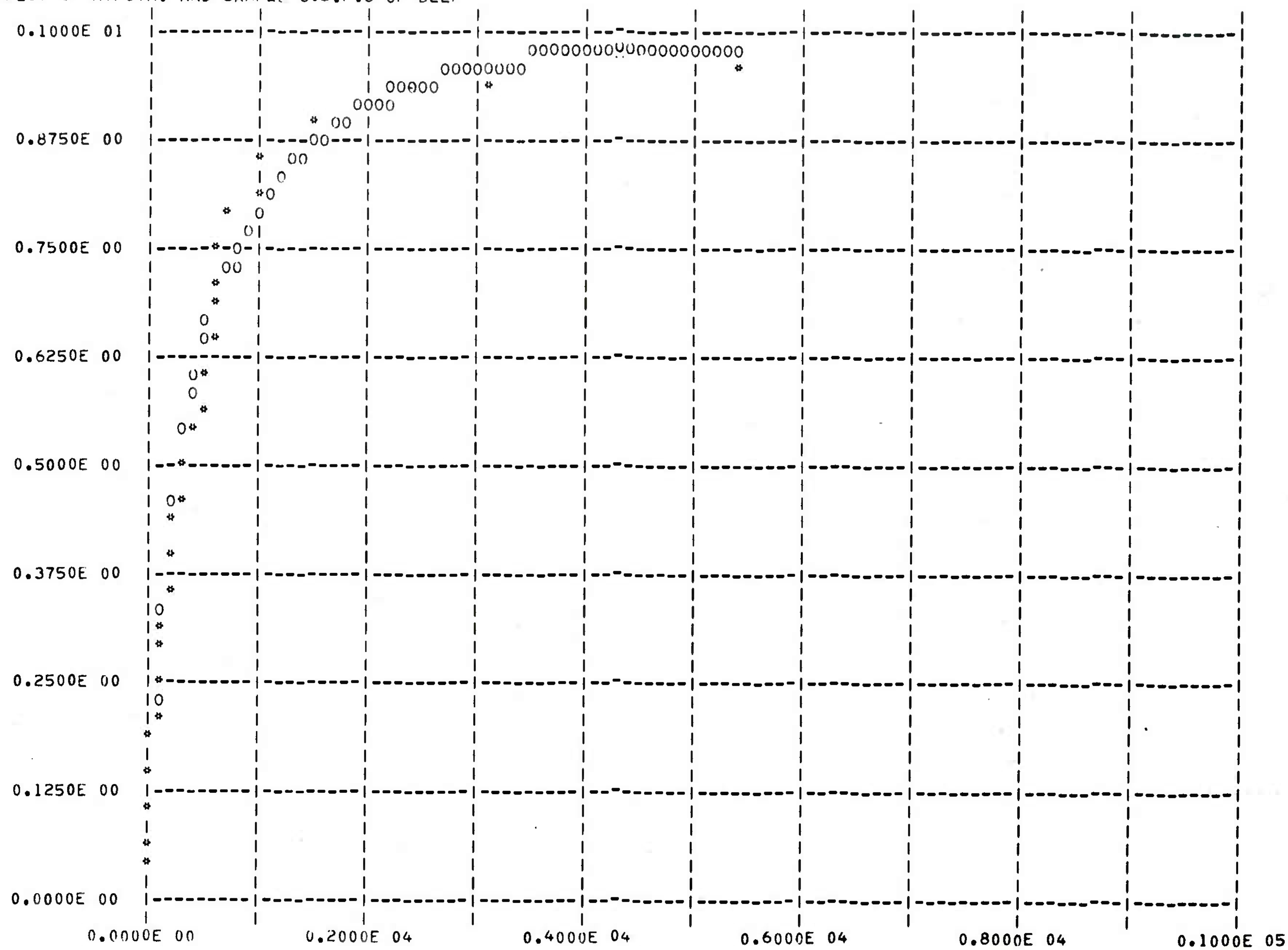
ZERO VALUES ENCOUNTERED PRECLUDES THE CALCULATION OF M AXIMUM LIKELIHOOD ESTIMATES. MATCHING MOM. PARAMS. ARE KEPT.

INDEX	DELP,KS1	CDF	DSTAR	SSTAR
1	0.0000	-0.0000	0.0370	0.0370
2	0.0000	-0.0000	0.0741	0.1111
3	0.0000	-0.0000	0.1111	0.2222
4	30.0000	0.1659	0.0547	0.2770
5	39.0000	0.1922	0.0441	0.3210
6	106.0000	0.3278	0.1426	0.4636
7	124.0000	0.3546	0.1323	0.5960
8	127.0000	0.3588	0.0995	0.6955
9	143.0000	0.3802	0.0839	0.7794
10	174.0000	0.4175	0.0841	0.8635
11	190.0000	0.4349	0.0645	0.9280
12	197.0000	0.4422	0.0347	0.9627
13	296.0000	0.5284	0.0839	1.0467
14	302.0000	0.5328	0.0513	1.0980
15	436.0000	0.6156	0.0971	1.1951
16	458.0000	0.6268	0.0713	1.2663
17	476.0000	0.6356	0.0430	1.3094
18	556.0000	0.6710	0.0414	1.3508
19	584.0000	0.6822	0.0215	1.3723
20	644.0000	0.7042	0.0365	1.4088
21	648.0000	0.7056	0.0722	1.4810
22	713.0000	0.7268	0.0880	1.5690
23	990.0000	0.7963	0.0556	1.6246
24	1017.0000	0.8017	0.0872	1.7119
25	1479.0000	0.8702	0.0558	1.7676
26	3115.0000	0.9609	0.0350	1.8026
27	5388.0000	0.9895	0.0265	1.8292

SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F. USING THE F-S TEST S STAR = 1.8292

LILLIEFORS (K-S) TEST SHOWS SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F.
TEST STATISTIC 0.1426 IS LESS THAN THE CRITICAL VALUE 0.1665 WITH A RISK OF 0.0500

PLOT OF HYPOTH. AND SAMPLE C.D.F.S OF DELP



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TEST DATA FOR 8 IN. HDW. M188E1 CHG.--SET C4,COLD SOAK (-50 DEG F)

SAMPLE =  23  F-S CRITICAL VALUE =      2.337     F-S LEVEL OF RISK =     0.0500     DEG. OF POLY. REGRES. =  3

   AVG PMAX,KSI   AVG DELP,PSI   AVG TMDEL,MS
      37.2869          467.3          153.7

  SD:  PMAX,KSI        DELP,PSI       DELAY,MS
        0.5155        274.3997        35.2995

CORRELATION MATRIX:

          PMAX            DELP           DELAY
        1.0000         -0.0499          0.3734
       -0.0499          1.0000          0.0773
        0.3734          0.0773          1.0000

ANALYSIS BASED ON SIMPLE LINEAR REGRESSION OF PMAX ON DELP:

SLOPE =  -0.00009 KSI/PSI     STD. ERR. OF EST. OF PMAX GIVEN DELP =   0.52696 KSI

STD. DEV. OF SLOPE =   0.00041     95% CONF. LIMITS:    -0.00095   0.00076     T-STATISTIC (CRITICAL) =   2.07965

PARTIAL CORRELATION COEFS. WITH DEPENDENCE OF PMAX ON DELP AND DELAY ASSUMED

 DELP, GIVEN DT DT, GIVEN DELP
      -0.0852          0.3789

FRACTION OF VARIANCE DUE TO DELP =           0.0073

FRACTION OF VARIANCE DUE TO DELAY =           0.1436

    I    NORM. COEF.          XBAR(I)        SIGMA(I)  UNIT RESIDUAL
    1   -0.000000E-01   4.672607E 02   2.744001E 02   0.000000E-01
    2    2.968628E 00   2.903544E 05   2.848810E 05  -1.735842E-06
    3   -7.212316E 00   2.076444E 08   2.754061E 08   3.441966E-05
    4    4.369547E 00   3.728694E 01   5.154914E-01  -1.106674E-04

DIMENSIONALIZED COEFFICIENTS
X( 1)= 3.67721200E 01
X( 2)= 5.57689700E-03
X( 3)=-1.30506600E-05
X( 4)= 8.17869800E-09
     RES. M.S.      R-SQUARED
   2.680200E-01      0.128926

REGRESSION COEFS. FOR PMAX ON DELP:

        ZEROTH         FIRST            2ND            3RD
    0.36772E 02   0.55769E-02   -0.13051E-04    0.81787E-08

FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION:        0.12893
```

DISTRIBUTION OF RESIDUALS FROM PMAX-DELP REGRESSION

INDEX	RESID,KSI	MED RANK	NORM D.F.	DSTAR	SSTAR
1	-0.7785	0.0417	0.0528	0.0528	0.0528
2	-0.7470	0.0833	0.0602	0.0267	0.0795
3	-0.6838	0.1250	0.0776	0.0528	0.1324
4	-0.4234	0.1667	0.1894	0.0589	0.1913
5	-0.3683	0.2083	0.2220	0.0481	0.2394
6	-0.2904	0.2500	0.2730	0.0556	0.2950
7	-0.2651	0.2917	0.2908	0.0300	0.3250
8	-0.2194	0.3333	0.3241	0.0237	0.3486
9	-0.1721	0.3750	0.3602	0.0311	0.3797
10	-0.1531	0.4167	0.3752	0.0596	0.4393
11	-0.0388	0.4583	0.4678	0.0331	0.4724
12	-0.0361	0.5000	0.4700	0.0517	0.5241
13	0.0098	0.5417	0.5081	0.0571	0.5812
14	0.0134	0.5833	0.5111	0.0976	0.6788
15	0.1136	0.6250	0.5933	0.0589	0.7377
16	0.2127	0.6667	0.6708	0.0249	0.7625
17	0.2240	0.7083	0.6792	0.0599	0.8225
18	0.3229	0.7500	0.7489	0.0337	0.8561
19	0.3244	0.7917	0.7499	0.0762	0.9323
20	0.3329	0.8333	0.7555	0.1141	1.0464
21	0.5798	0.8750	0.8859	0.0271	1.0735
22	0.8894	0.9167	0.9677	0.0547	1.1282
23	1.1537	0.9583	0.9918	0.0352	1.1634

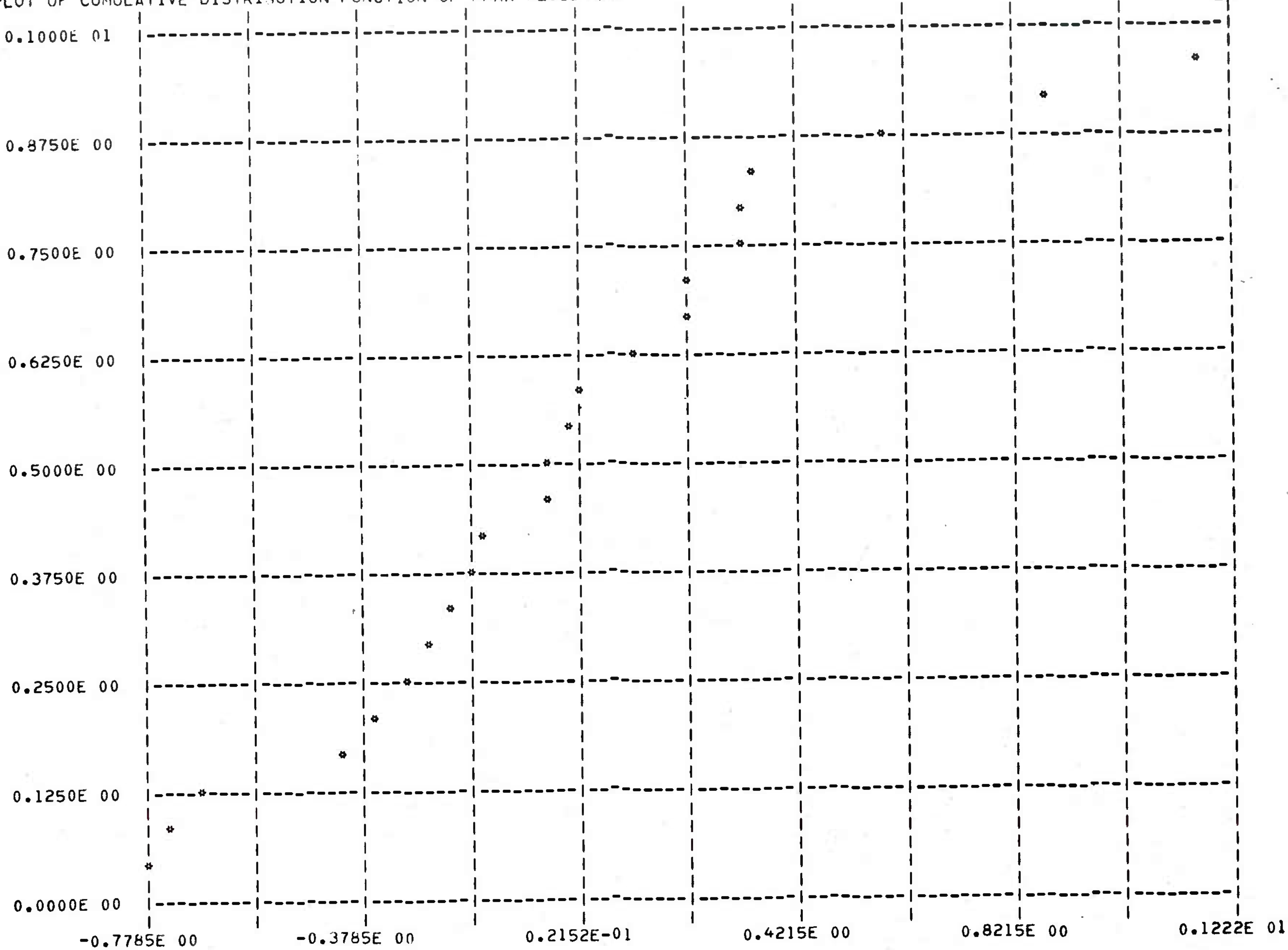
RESIDUALS ARE PLAUSIBLY GAUSSIAN.

SSTAR IN F-S TEST,   1.1634 IS LESS THAN THE CRITICAL VALUE,   2.3374

LILLIEFORS (K-S) TEST SHOWS RESIDUALS ARE GAUSSIAN.
TEST STATISTIC   0.1141 IS LESS THAN THE CRITICAL VALUE   0.1798 WITH A RISK OF   0.0500

MEAN AND STD. DEV. OF RESIDUALS (KSI):   0.00002   0.48112

PLOT OF CUMULATIVE DISTRIBUTION FUNCTION OF PMAX RESIDUALS



I	NORM. COEF.	XBAR(I)	SIGMA(I)	UNIT RESIDUAL
1	-0.000000E-01	1.536522E 02	3.529950E 01	0.000000E-01
2	1.140126E 01	2.480087E 04	1.164159E 04	-3.076172E-05

3	-2.149130E 01	4.199822E 06	3.054946E 06	6.371963E-07
4	1.063285E 01	3.728694E 01	5.154914E-01	-9.620146E-06

DIMENSIONALIZED COEFFICIENTS
X(1)= 2.77705300E 01
X(2)= 1.66496600E-01
X(3)=-9.51638200E-04
X(4)= 1.79418400E-06

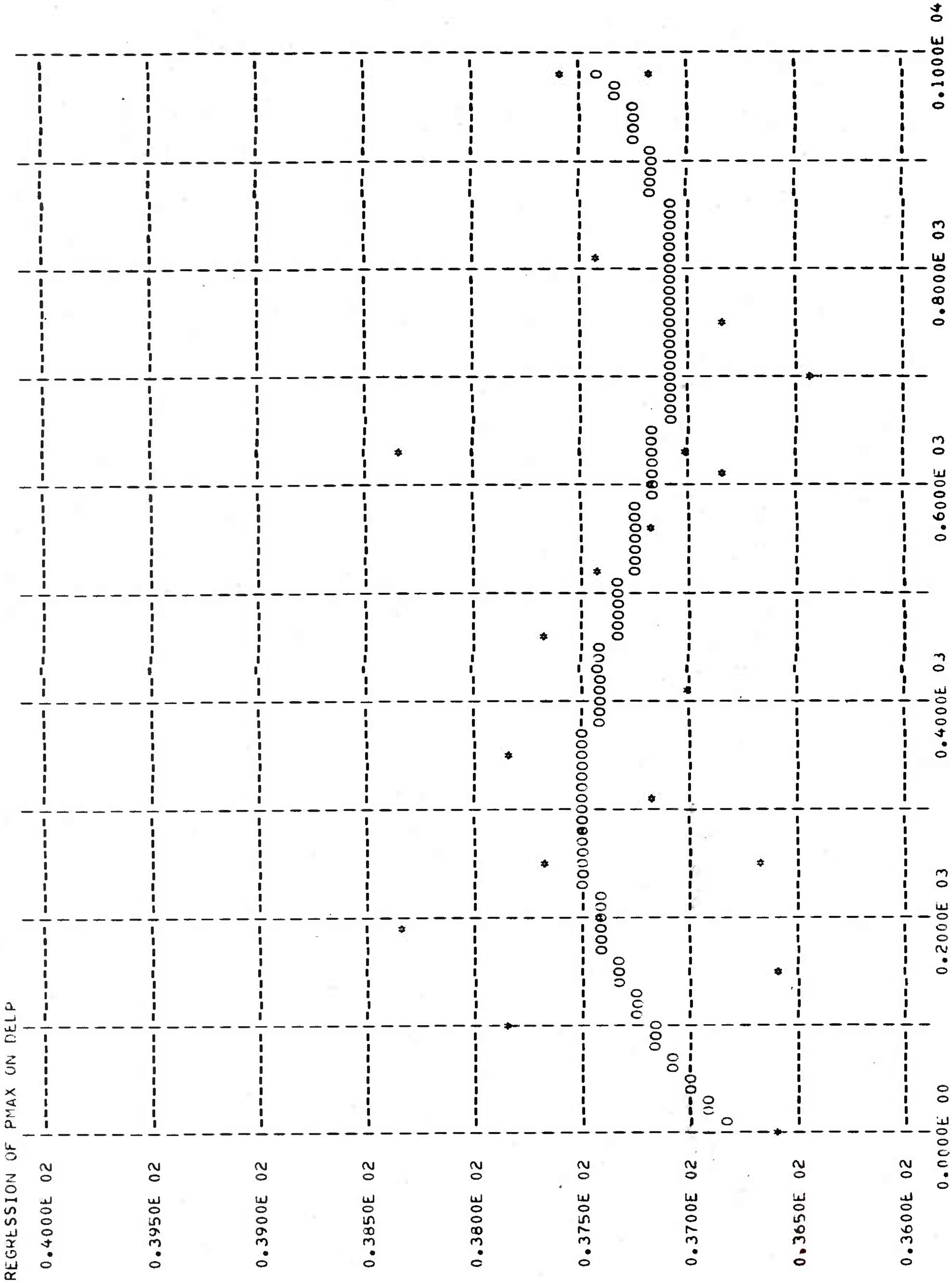
RES. M.S.	R-SQUARED
2.445485E-01	0.205209

REGRESSION COEFS. FOR PMAX ON DELAY:

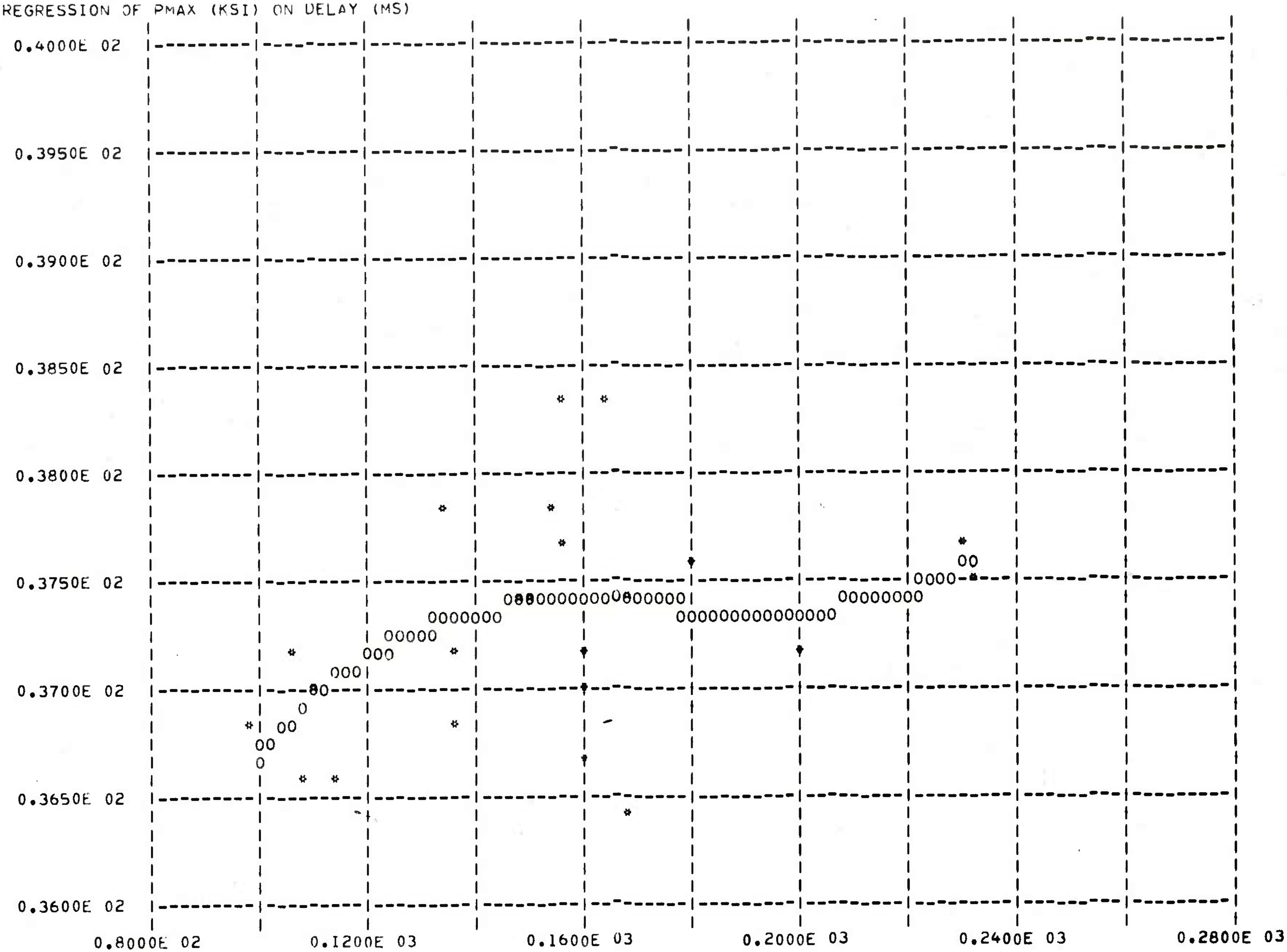
ZEROTH	FIRST	2ND	3RD
0.27771E 02	0.16650E 00	-0.95164E-03	0.17942E-05

FRACTION OF VARIANCE EXPLAINED BY THE REGRESSION: 0.20521

INDEX	MED RANK	PMAX,KSI	DELP,PSI	DELAY,MS
1	0.0417	36.4000	0.00	98.00
2	0.0833	36.6000	104.00	106.00
3	0.1250	36.6000	152.00	108.00
4	0.1667	36.7000	186.00	110.00
5	0.2083	36.8000	204.00	115.00
6	0.2500	36.8000	246.00	135.00
7	0.2917	37.0000	250.00	136.00
8	0.3333	37.0000	281.00	137.00
9	0.3750	37.2000	310.00	148.00
10	0.4167	37.2000	348.00	151.00
11	0.4583	37.2000	415.00	154.00
12	0.5000	37.2000	464.00	155.00
13	0.5417	37.4000	525.00	157.00
14	0.5833	37.4000	560.00	160.00
15	0.6250	37.4000	600.00	160.00
16	0.6667	37.5000	615.00	160.00
17	0.7083	37.6000	628.00	164.00
18	0.7500	37.7000	634.00	168.00
19	0.7917	37.7000	704.00	169.00
20	0.8333	37.8000	746.00	180.00
21	0.8750	37.8000	814.00	201.00
22	0.9167	38.3000	976.00	230.00
23	0.9583	38.3000	985.00	232.00


REGRESSION OF PMAX ON DELP
0.4000E 02
0.3950E 02
0.3900E 02
0.3850E 02
0.3800E 02
0.3750E 02
0.3700E 02
0.3650E 02
0.3600E 02
0.0000E 00
0.2000E 03
0.4000E 03
0.6000E 03
0.8000E 03
0.1000E 04

REGRESSION OF PMAX (KSI) ON DELAY (MS)



PARAMETER ESTIMATES OF THE C.D.F. OF DELP OBTAINED BY MATCHING MOMENTS

A = 1.7586 ETA = 524.7942 DF = -0.00014 ETA = 0.00078 KOUNT = 21

PROXIMATE STD. DEV. OF FSTIMATE OF BETA = 0.3685

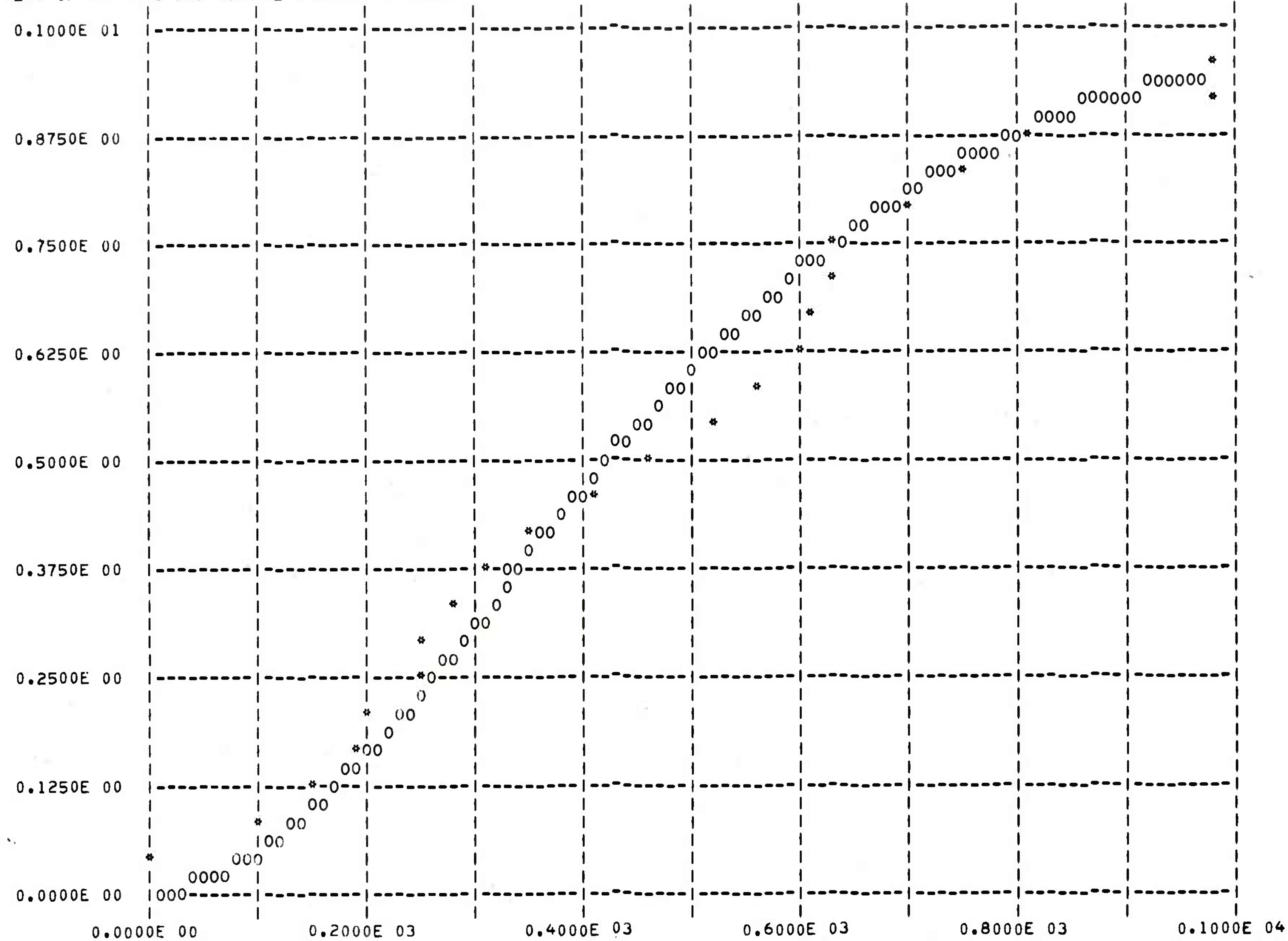
ZERO VALUES ENCOUNTERED PRECLUDES THE CALCULATION OF MAXIMUM LIKELIHOOD ESTIMATES. MATCHING MOM. PARAMS. ARE KEPT.

INDEX	DELP,KSI	CDF	DSTAR	SSTAR
1	0.0000	-0.0000	0.0435	0.0435
2	104.0000	0.0564	0.0306	0.0740
3	152.0000	0.1070	0.0235	0.0975
4	186.0000	0.1490	0.0249	0.1224
5	204.0000	0.1729	0.0445	0.1669
6	246.0000	0.2319	0.0290	0.1959
7	250.0000	0.2377	0.0666	0.2625
8	281.0000	0.2835	0.0643	0.3268
9	310.0000	0.3271	0.0642	0.3910
10	348.0000	0.3847	0.0501	0.4411
11	415.0000	0.4841	0.0493	0.4904
12	464.0000	0.5531	0.0748	0.5652
13	525.0000	0.6324	0.1106	0.6758
14	560.0000	0.6740	0.1088	0.7847
15	600.0000	0.7179	0.1092	0.8939
16	615.0000	0.7333	0.0812	0.9750
17	628.0000	0.7462	0.0506	1.0256
18	634.0000	0.7520	0.0306	1.0562
19	704.0000	0.8129	0.0303	1.0865
20	746.0000	0.8437	0.0258	1.1124
21	814.0000	0.8851	0.0279	1.1403
22	976.0000	0.9491	0.0360	1.1763
23	985.0000	0.9515	0.0485	1.2248

SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F. USING THE F-S TEST S STAR = 1.2248

LILLIEFORS (K-S) TEST SHOWS SAMPLE IS PLAUSIBLE FROM THE HYPOTH. C.D.F.
TEST STATISTIC 0.1106 IS LESS THAN THE CRITICAL VALUE 0.1798 WITH A RISK OF 0.0500

PLOT OF HYPOTH. AND SAMPLE C.D.F.S OF DELP



```
C     CONSTRAINED QUADRATIC REGRESSION
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TITLE(20),DATA(400,3),Y(400),X1(400),X2(400),XMS(4),
     1 H(4),C0(4),C1(4),C2(4),R(4),S(4)
      EQUIVALENCE (DATA(1,1),Y(1)),(DATA(1,2),X1(1)),(DATA(1,3),X2(1))
      COMMON/GCOM/R0,R1
    1 CONTINUE
      READ (5,10,END=3) TITLE
   10 FORMAT(20A4)
      WRITE (6,12) TITLE
   12 FORMAT(1H1,20A4)
      READ (5,14) NSAMP,A0,A1,A2,A3
   14 FORMAT(1I3,7X,4F10.0)
      WRITE (6,114) A0,A1,A2,A3
  114 FORMAT(1H0,8X,2HA0,8X,2HA1,8X,2HA2,8X,2HA3/1H ,4F10.4)
      X=0.0D0
      DX=0.1D0
      WRITE(6,13) NSAMP
   13 FORMAT(1H0,'NSAMP =',I3/1H0,9X,1HY,8X,2HX1,8X,2HX2)
C
C     READ DATA ARRAYS
C
      DO 15 I=1,NSAMP
      READ (5,18) Y(I),X1(I),X2(I)
   18 FORMAT(3F10.0)
      WRITE (6,17) Y(I),X1(I),X2(I)
   17 FORMAT(1H ,3F10.2)
   15 CONTINUE
C
C     CALCULATE R AND S COEFFICIENTS FROM THE DATA
C
      DO 20 J=1,4
      R(J)=0.0D0
      S(J)=0.0D0
   20 CONTINUE
      S0=0.0D0
      DO 22 I=1,NSAMP
      S0=S0+Y(I)
      DO 24 J=1,4
      R(J)=R(J)+X1(I)**J
      IF(J.GT.2) GO TO 21
      S(J)=S(J)+X1(I)**J*Y(I)
   21 CONTINUE
   24 CONTINUE
   22 CONTINUE
      R0=NSAMP
      R1=R(1)
C
C     CALCULATE ELEMENTS OF THE A MATRIX
C
      A11=R(2)-R(1)**2/R0
      A12=R(3)-R(1)*R(2)/R0
      A21=A12
      A22=R(4)-R(2)**2/R0
      B1=S(1)-R(1)*S0/R0
      B2=S(2)-R(2)*S0/R0
C
```

```
C     CASE 1, NO CONSTRAINTS:  LAMBDA1(ELS1)=LAMBDA2(ELS2) = 0.
C
      DENOM=A11*A22-A12*A21
      C1(1)=(A22*B1-A12*B2)/DENOM
      C2(1)=(A11*B2-A21*B1)/DENOM
      C0(1)=S0/R0-C1(1)*R(1)/R0-C2(1)*R(2)/R0
      CALL GRANGE(C0(1),C1(1),C2(1),0.0D0,0.0D0,NSAMP,400,X1,Y,H(1),
     1 XMS(1))
C
C     CASE 2, C1=0, C2.NE.0 (LAMBDA2=0)
C
      C1(2)=0.0D0
      C2(2)=B2/A22
      ELS1=2.0D0*(C2(2)*A12-B1)/R0
      ELS2=0.0D0
      C0(2)=S0/R0-C2(2)*R(2)/R0
      CALL GRANGE(C0(2),C1(2),C2(2),ELS1,ELS2,NSAMP,400,X1,Y,H(2),
     1 XMS(2))
C
C     CASE 3, C1.NE.0 (LAMBDA1=0). C2=0
C
      C2(3)=0.0D0
      C1(3)=B1/A11
      ELS1=0.0D0
      ELS2=(C1(3)*A21-B2)/R(1)
      C0(3)=S0/R0-C1(3)*R(1)/R0
      CALL GRANGE(C0(3),C1(3),C2(3),ELS1,ELS2,NSAMP,400,X1,Y,H(3),
     1 XMS(3))
C
C     CASE 4, C1 = C2 = 0. LAMBDA1.NE.0, LAMBDA2.NE.0.
C
      C1(4)=0.0D0
      C2(4)=0.0D0
      C0(4)=S0/R0
      ELS1=-2.0D0*B1/R0
      ELS2=-B2/R(1)
      CALL GRANGE(C0(4),C1(4),C2(4),ELS1,ELS2,NSAMP,400,X1,Y,H(4),
     1 XMS(4))
C
C     FIND A FEASIBLE CASE FOR WHICH THE LAGRANGIAN IS MINIMAL
C
      IF(C1(1).LT.0.0D0.OR.C2(1).LT.0.0D0) GO TO 48
      HMIN=H(1)
      GO TO 50
   48 CONTINUE
      IF(C1(2).LT.0.0D0.OR.C2(2).LT.0.0D0) GO TO 52
      IF(C1(3).LT.0.0D0.OR.C2(3).LT.0.0D0) GO TO 54
      HMIN=DMIN1(H(2),H(3),H(4))
      GO TO 50
   52 CONTINUE
C
C      OMIT CASE 2 AS FEASIBLE
C
      IF(C1(3).LT.0.0D0.OR.C2(3).LT.0.0D0) GO TO 56
      HMIN=DMIN1(H(3),H(4))
      GO TO 50
   56 HMIN=H(4)
      GO TO 50
```

```
   54 CONTINUE
C
C     CASE2 IS FEASIBLE; OMIT CASE 3
      HMIN=DMIN1(H(2),H(4))
   50 CONTINUE
C
C     WRITE HEADINGS FOR THE OUTPUT OF ALL CASES
C
      WRITE (6,28)
   28 FORMAT(1H0,10H  CASE NO.,13X,2HC0,13X,2HC1,13X,2HC2,5X,
     1 10HLAGRANGIAN,5X,10HMEAN SQRS.)
      DO 30 J=1,4
      IF(H(J).EQ.HMIN) JOPTI=J
      WRITE (6,32) J,C0(J),C1(J),C2(J),H(J),XMS(J)
   32 FORMAT(1H ,7X,I3,5D15.5)
   30 CONTINUE
      WRITE (6,34) JOPTI
   34 FORMAT(1H0,'THE OPTIMAL CASE UNDER CONSTRAINTS IS ',I3)
      GO TO 1
C
C     RETURN FOR PROCESSING OF ANOTHER DATA SET
C
    3 CONTINUE
      CALL EXIT
      STOP
      END

      SUBROUTINE GRANGE(C0,C1,C2,ELS1,ELS2,N,NDIM,X,Y,H,XMS)
C
C     THIS PROGRAM CALCULATES THE LAGRANGIAN AND MEAN SQUARED
C     ERROR FOR A CONSTRAINED QUADRATIC REGRESSION OF Y ON X.
C     CONSTRAINTS ARE:  POSITIVE Y(X) AND POSITIVE SLOPE EVERYWHERE.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(NDIM),Y(NDIM)
      COMMON/GCOM/R0,R1
      XMS=0.0D0
      H=0.0D0
      DO 1 I=1,N
      TEMP=(Y(I)-C0-C1*X(I)-C2*X(I)*X(I))**2
      XMS=XMS+TEMP
    1 CONTINUE
      H=XMS-ELS1*R0*C1-2.0D0*ELS2*R1*C2
      XMS=XMS/DFLOAT(N-3)
      RETURN
      END

$ENTRY
```

SECURED CARGO (HOT)--SET H1

A0	A1	A2	A3
0.0000	0.0000	0.0000	0.0000

NSAMP = 26

Y	X1	X2
42.80	2740.00	60.00
42.70	843.00	84.00
41.50	531.00	94.00
42.60	1886.00	51.00
43.00	813.00	87.00
43.50	641.00	123.00
41.00	226.00	88.00
41.20	889.00	104.00
42.00	903.00	90.00
42.40	68.00	102.00
43.00	650.00	81.00
42.50	1681.00	78.00
43.20	227.00	103.00
43.00	459.00	102.00
42.20	424.00	93.00
42.10	1661.00	78.00
42.60	744.00	98.00
43.00	2321.00	79.00
42.90	4008.00	57.00
42.30	2171.00	84.00
41.80	4200.00	66.00
41.30	2512.00	67.00
41.60	1944.00	104.00
41.60	733.00	103.00
42.60	1193.00	106.00
42.60	697.00	93.00

CASE NO.	C0	C1	C2	LAGRANGIAN	MEAN SQRS.
1	0.42393D 02	-0.48860D-04	0.65557D-08	0.11227D 02	0.48812D 00
2	0.42360D 02	0.00000D 00	-0.46592D-08	0.11233D 02	0.48840D 00
3	0.42377D 02	-0.22775D-04	0.00000D 00	0.11229D 02	0.48821D 00
4	0.42346D 02	0.00000D 00	0.00000D 00	0.11245D 02	0.48890D 00

THE OPTIMAL CASE UNDER CONSTRAINTS IS 4

SECURED CARGO (HOT)--SET H2

A0	A1	A2	A3
0.0000	0.0000	0.0000	0.0000

NSAMP = 21

Y	X1	X2
43.20	1591.00	60.00
42.90	736.00	87.00
43.60	525.00	81.00
42.40	3187.00	62.00
43.30	1865.00	72.00
41.80	1540.00	80.00
43.40	1760.00	60.00
43.20	6973.00	78.00
42.00	1883.00	46.00
41.40	2020.00	59.00
41.40	2098.00	63.00
43.20	92.00	110.00
42.30	341.00	83.00
41.80	344.00	75.00
42.50	1067.00	92.00
42.20	1064.00	82.00
43.00	1847.00	85.00
42.90	270.00	110.00
40.80	313.00	108.00
42.40	1737.00	60.00
42.80	621.00	110.00

CASE NO.	C0	C1	C2	LAGRANGIAN	MEAN SQRS.
1	0.42586D 02	-0.15655D-03	0.34355D-07	0.10955D 02	0.60860D 00
2	0.42441D 02	0.00000D 00	0.13422D-07	0.11090D 02	0.61611D 00
3	0.42396D 02	0.68355D-04	0.00000D 00	0.11273D 02	0.62630D 00
4	0.42500D 02	0.00000D 00	0.00000D 00	0.11480D 02	0.63778D 00

THE OPTIMAL CASE UNDER CONSTRAINTS IS 2

```
C     ************************************RISK M110A1*************************************
C
C     PROGRAM TO CALCULATE THE PROBABILITY THAT THE MAX CHAMBER
C     PRESSURE, PMAX, EXCEEDS A CRITICAL VALUE
C
      DIMENSION TITLE(20)
    1 CONTINUE
C
C     READ DESCRIPTIVE ALPHAMERIC TITLE
C
      READ (5,10,END=3) TITLE
   10 FORMAT(20A4)
      WRITE (6,12) TITLE
   12 FORMAT(1H1,20A4)
      READ (5,14) A0,A1,A2,SIGZ,BETA,ETA,PMCRIT,EPS
   14 FORMAT(7F10.0,1E10.3)
      WRITE (6,16) A0,A1,A2,SIGZ,BETA,ETA,PMCRIT,EPS
   16 FORMAT(1H0,10X,2HA0,10X,2HA1,10X,2HA2,4X,8HSIGZ,KSI,8X,4HBETA,
     1 5X,7HETA,KSI,2X,10HPMCRIT,KSI,9X,3HEPS/1H ,7F12.5,E12.3)
C
C     CALCULATE STEP SIZE FOR NUMERICAL INTEGRATION AND THE
C     INITIAL VALUE OF THE INDEPENDENT VARIABLE, DELP (KSI)
C
      XLIM=50.0E0
      H=AMIN1(0.002*SIGZ,0.002*ETA)
      HO2=H/2.0E0
      ALNE=-ALOG(EPS)
      XN=ETA*(ALNE)**(1.0E0/BETA)
      IF(XN.GT.XLIM) XN=XLIM
      Y=PMCRIT
   20 CONTINUE
C
C     START INTEGRATION AT X=0.
C
      X1=H
   22 CONTINUE
      WRITE (6,21) X1,XN,H
   21 FORMAT(1H0,'LIMITS OF INTEGRATION ON DELP (KSI) ARE:',2F12.5,
     1 4X,'STEP SIZE IS ',F12.5)
C
C     START INTEGRATION
C
      CALL GINT(Y,A0,A1,A2,SIGZ,BETA,ETA,X1,G1,GL1)
      RISK=GL1
      FY=G1
      X=X1
   24 CONTINUE
      X=X+H
      CALL GINT(Y,A0,A1,A2,SIGZ,BETA,ETA,X,G2,GL2)
      RISK=RISK+GL1+GL2
      FY=FY+G1+G2
      G1=G2
      GL1=GL2
      IF(X.LE.XN) GO TO 24
      FY=FY*HO2
      RISK=RISK*HO2
      RISK2=1.0E0-FY
```

```
      ERR=(GL1+GL2)*HO2
      WRITE (6,30) RISK,FY,RISK2,ERR
   30 FORMAT(1H0,20H RISK PMAX.GT.PMCRIT,6X,14HC.D.F.(PMCRIT),
     1 10X,10H1 - C.D.F.,8X,12HSTEP IN RISK/1H ,4E20.5)
      GO TO 1
    3 CONTINUE
      CALL EXIT
      STOP
      END
C     ************************* GINT ****************************************

      SUBROUTINE GINT(Y,A0,A1,A2,SIGZ,BETA,ETA,X,G,GL)
C
C     SUBROUTINE EVALUATES THE INTEGRAND IN THE M110A1 RISK ASSESSMENT
C
      FX=(BETA/ETA)*(X/ETA)**(BETA-1.0E0)* EXP(-(X/ETA)**BETA)
      PMBAR=A0+A1*X+A2*X*X
      IF(PMBAR.LT.0.0E0) PMBAR=0.0E0
      Z=Y-PMBAR
      ARGZ=Z/SIGZ
      IF(ARGZ.LT.-5.0E0) GO TO 1
      IF(ARGZ.GT.5.0E0) GO TO 2
      FZ=SNORM(ARGZ)
      IF(FZ.LT.0.0E0) GO TO 4
      IF(FZ.GT.1.0E0) FZ=1.0E0
      GO TO 3
    1 FZ=0.0E0
      GO TO 3
    2 FZ=1.0E0
    3 FZ1=1.0E0-FZ
      G=FX*FZ
      GL=FX*FZ1
      RETURN
    4 CONTINUE
      WRITE (6,5)
    5 FORMAT(1H0,'ERROR.  FZ IS NEGATIVE')
      CALL EXIT
      STOP
      END

$ENTRY
```

RISK OF A CAT. FAILURE IN THE M110A1 SP HOW USING THE M188E1 CHG AT 145F W/ R.H.

A0	A1	A2	SIGZ,KSI	BETA	ETA,KSI	PMCRIT,KSI	EPS
41.88800	0.11657	0.06800	0.58390	1.08100	1.33100	53.00000	0.100E-05

LIMITS OF INTEGRATION ON DELP (KSI) ARE: 0.00117 15.10415 STEP SIZE IS 0.00117

RISK PMAX.GT.PMCRIT	C.D.F.(PMCRIT)	1 - C.D.F.	STEP IN RISK
0.22172E-04	0.99863E 00	0.13714E-02	0.11535E-08

```
RISK OF A FAILURE IN THE M110A1 SP HOW USING THE M188E1 CHG AT 145 F W/O R. H.

       A0          A1          A2    SIGZ,KSI        BETA     ETA,KSI  PMCRIT,KSI         EPS
 42.22301    -0.11881     0.03762     0.75510     1.22200     1.57000    53.00000   0.100E-05

LIMITS OF INTEGRATION ON DELP (KSI) ARE:     0.00151    13.46160    STEP SIZE IS     0.00151

 RISK PMAX.GT.PMCRIT      C.D.F.(PMCRIT)          1 - C.D.F.        STEP IN RISK
         0.00000E 00         0.99889E 00         0.11116E-02         0.00000E 00
```

RISK OF FAILURE IN THE M110A1 USING THE M188E1 CHG AT -50 F W/ R. H.

A0	A1	A2	SIGZ,KSI	BETA	ETA,KSI	PMCRIT,KSI	EPS
36.79800	0.34983	0.03018	0.62270	0.62110	0.46880	53.00000	0.100E-06

LIMITS OF INTEGRATION ON DELP (KSI) ARE: 0.00094 41.19250 STEP SIZE IS 0.00094

RISK PMAX.GT.PMCRIT	C.D.F.(PMCRIT)	1 - C.D.F.	STEP IN RISK
0.63789E-04	0.98399E 00	0.16007E-01	0.22782E-10

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